Spatial Problem Solving

with Cuisenaire® Rods

by

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Robert E. Willcutt

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COVERING DESIGNS WITH ONE ROD OF EACH COLOR

Use exactly one rod of each of the ten colors to cover this design. Trace your solution and record the color names to show how you placed the rods. There is more than one correct way.
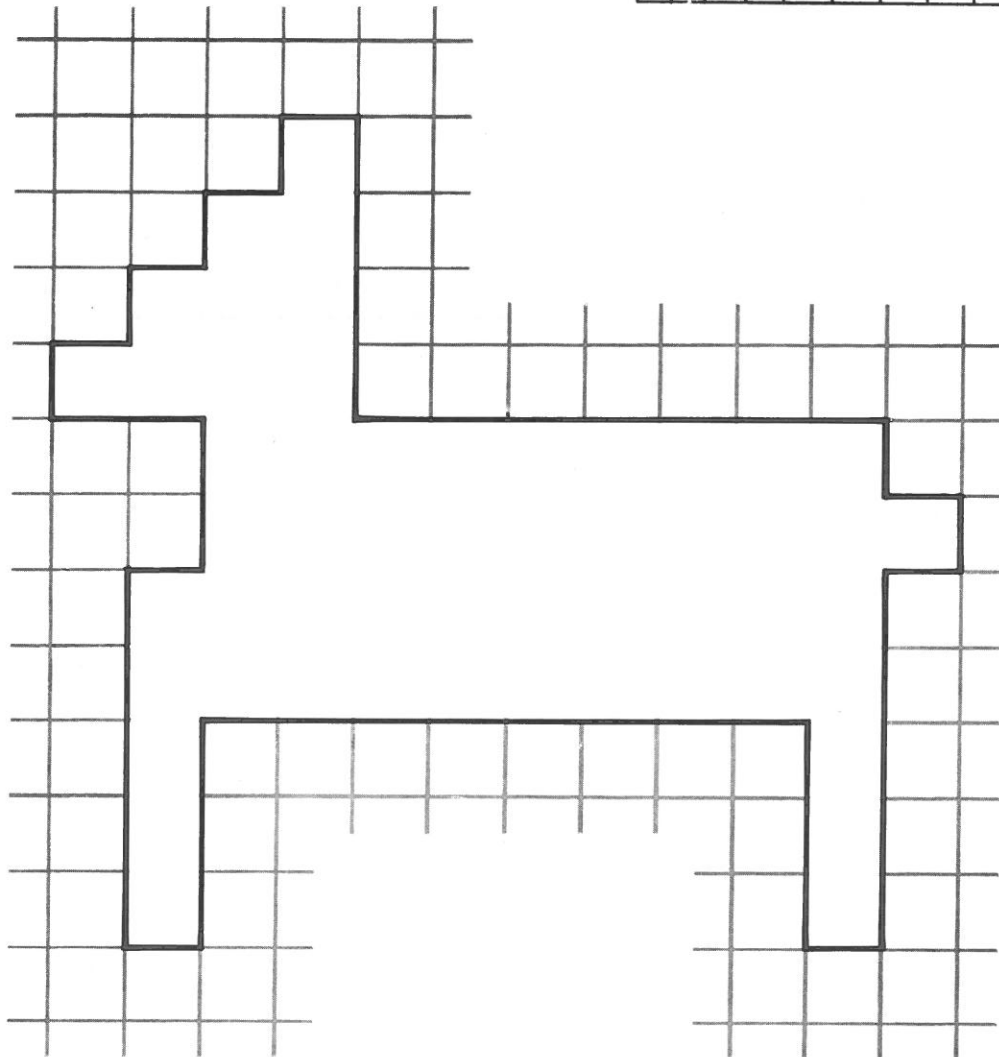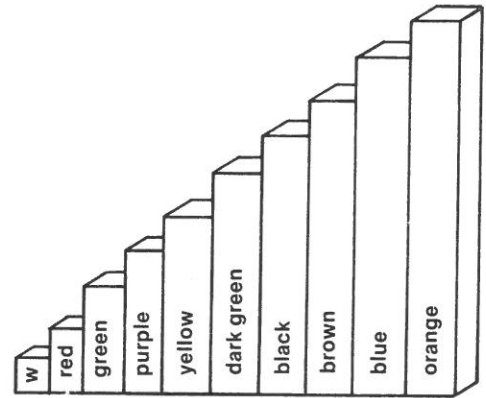
COVERING DESIGNS WITH ONE ROD OF EACH COLOR

Use exactly one rod of each of the ten colors to cover each design. Trace your solutions and record the color names to show how you placed the rods.

1)

2)
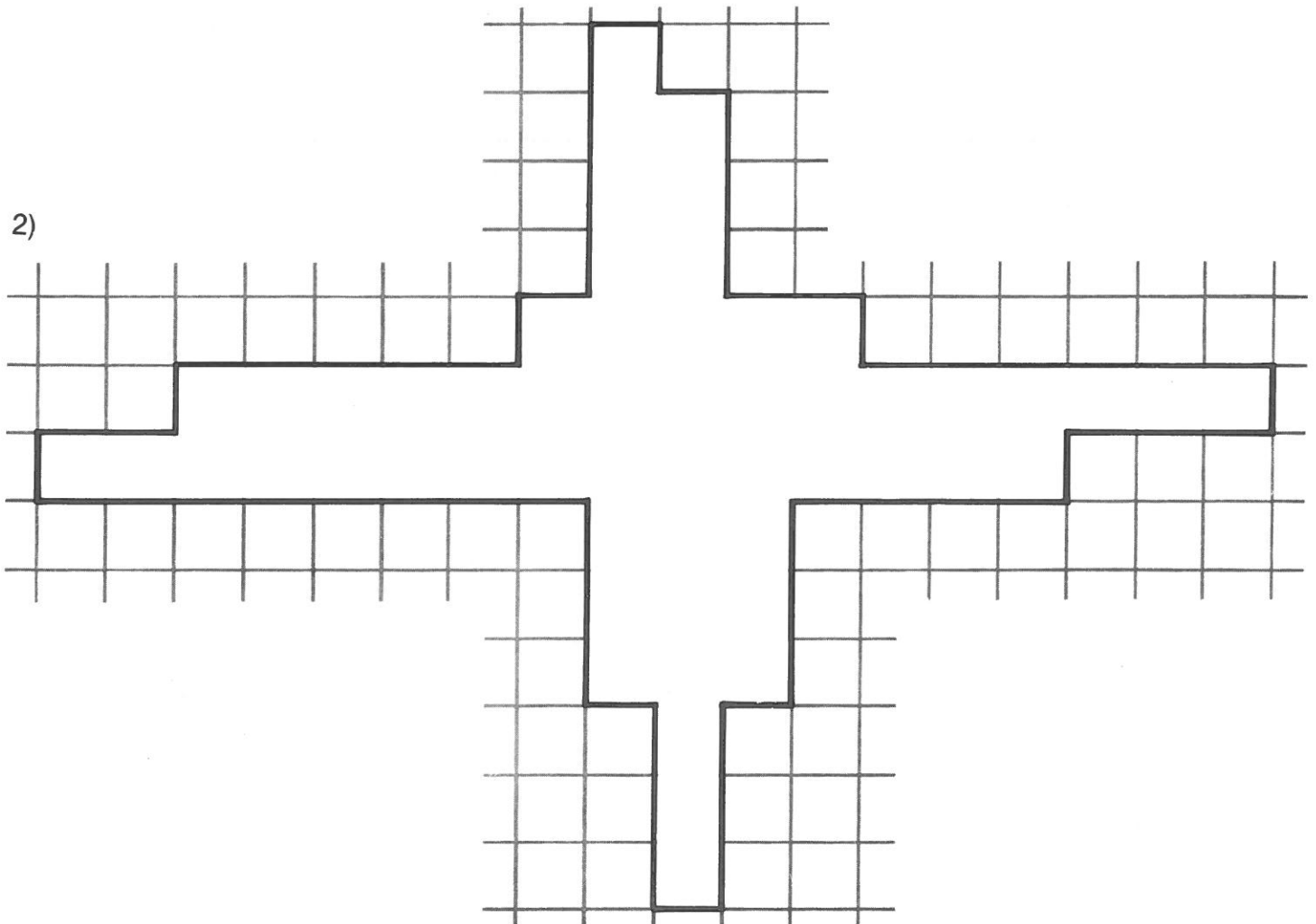
COVERING DESIGNS WITH A GIVEN SET OF RODS

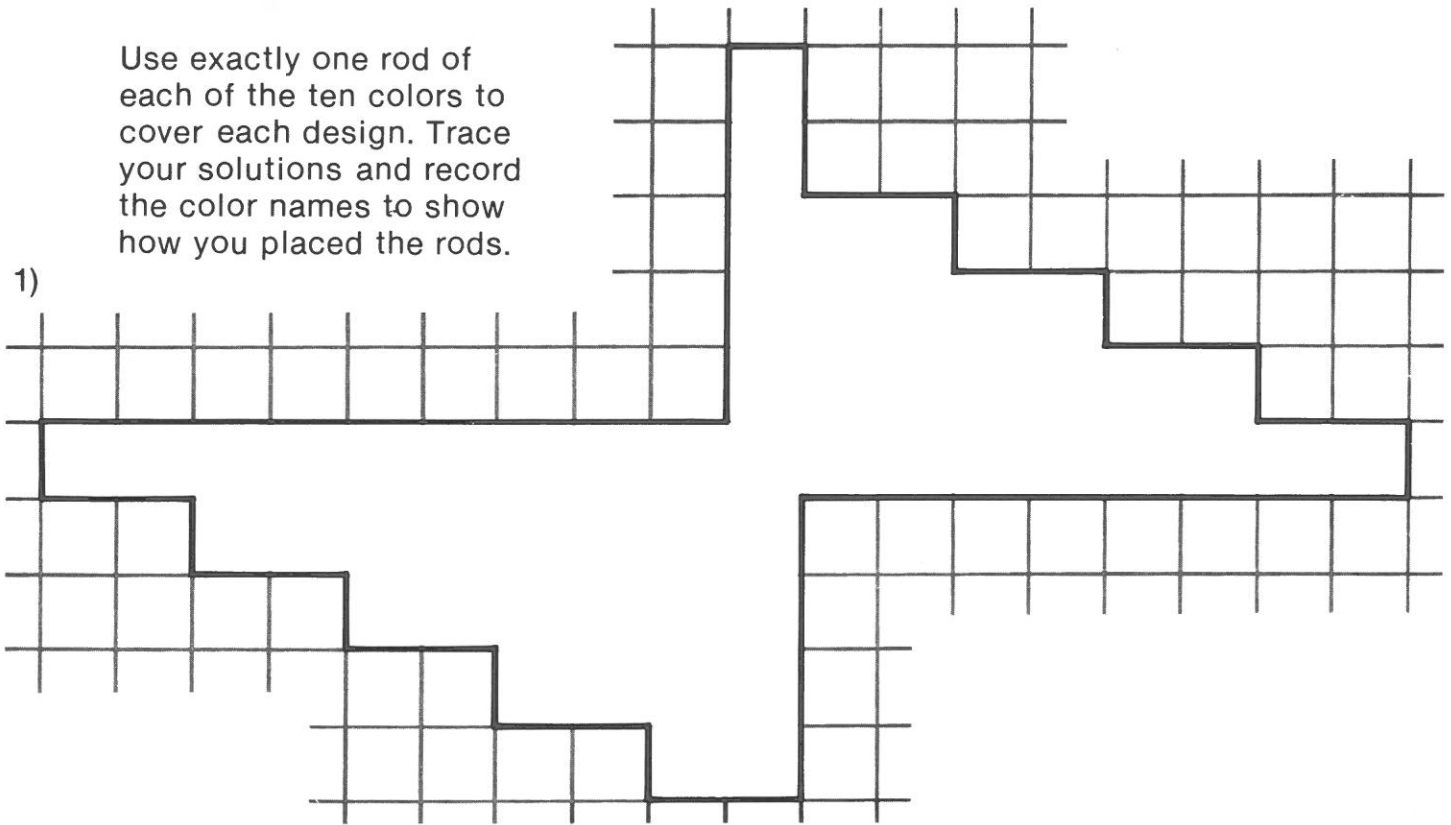Use the given rods to cover these designs. Record the color names on the designs to show your solutions. Can these be done in more than one way?

1)

brown   black   dark   yellow   purple   green   red   white
green

2)

blue   black   dark   yellow   purple   green   red   white
green

COVERING DESIGNS WITH A GIVEN SET OF RODS

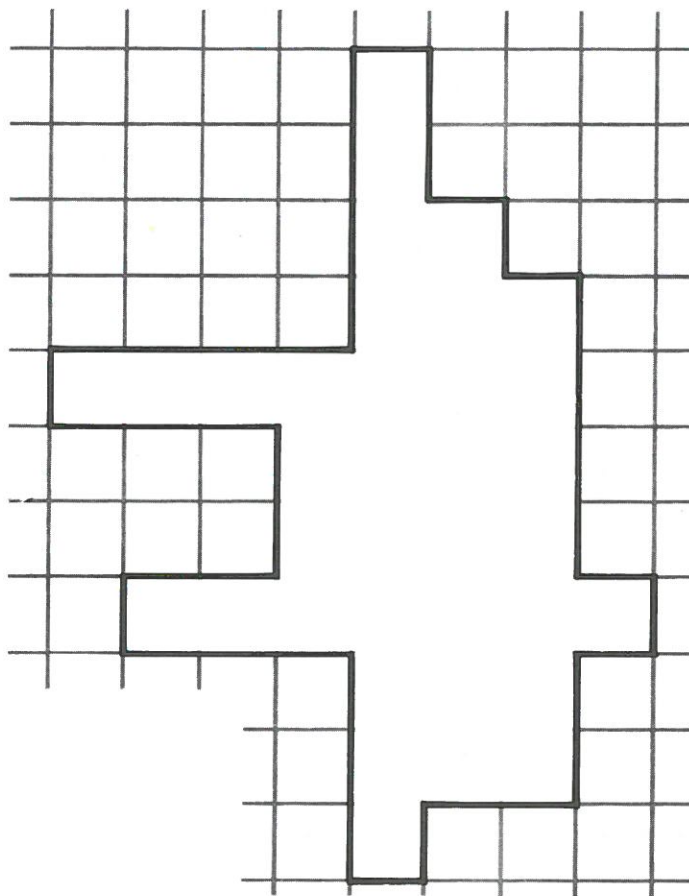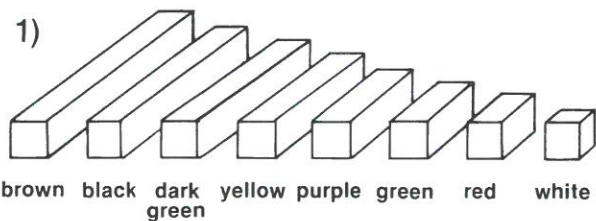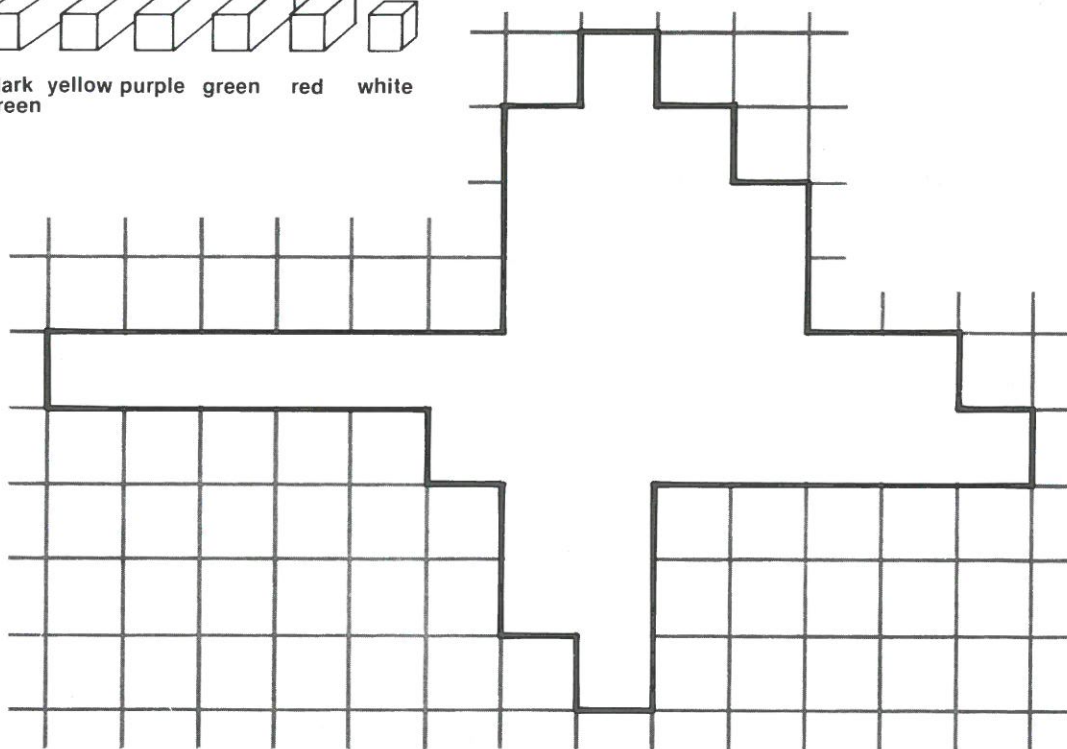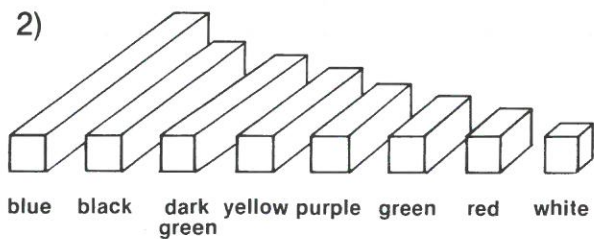Use the given rods to cover these designs. Record the color names on the designs to show your solutions. Can these be done in more than one way?
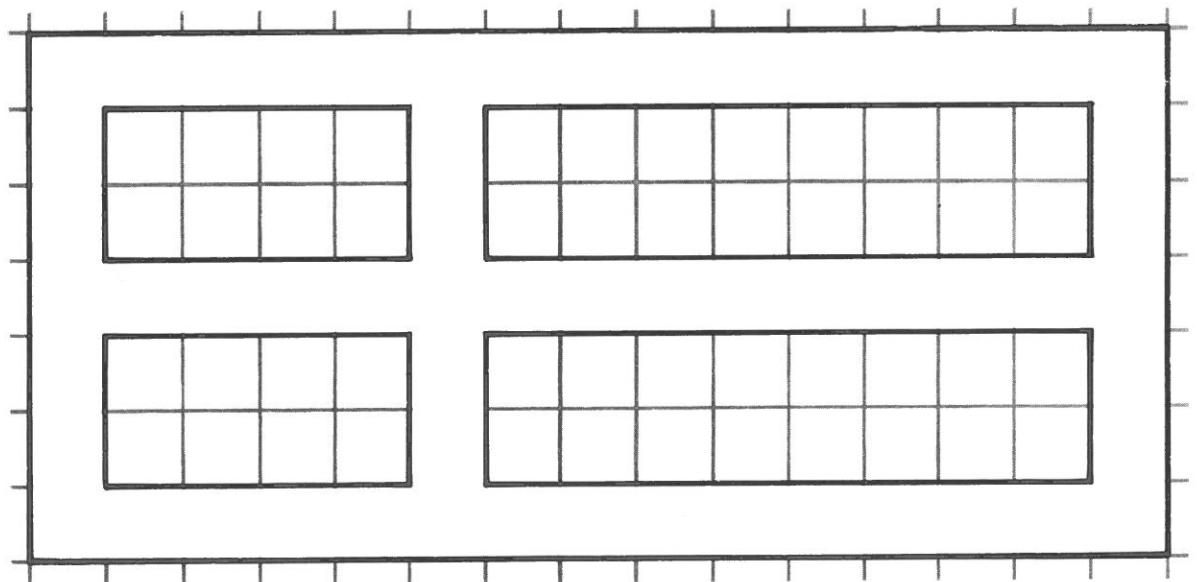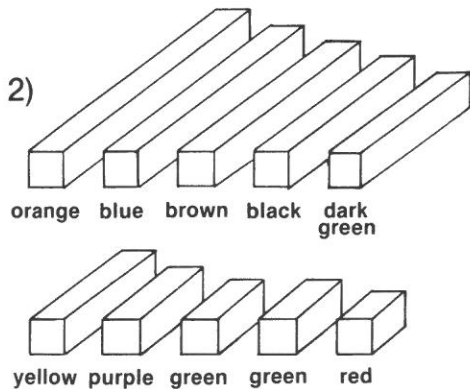
1)

orange   blue   brown   black   dark
                                 green

yellow   purple   green   green   red

2)

orange   blue   brown   black   dark
                                 green

yellow   purple   green   green   red
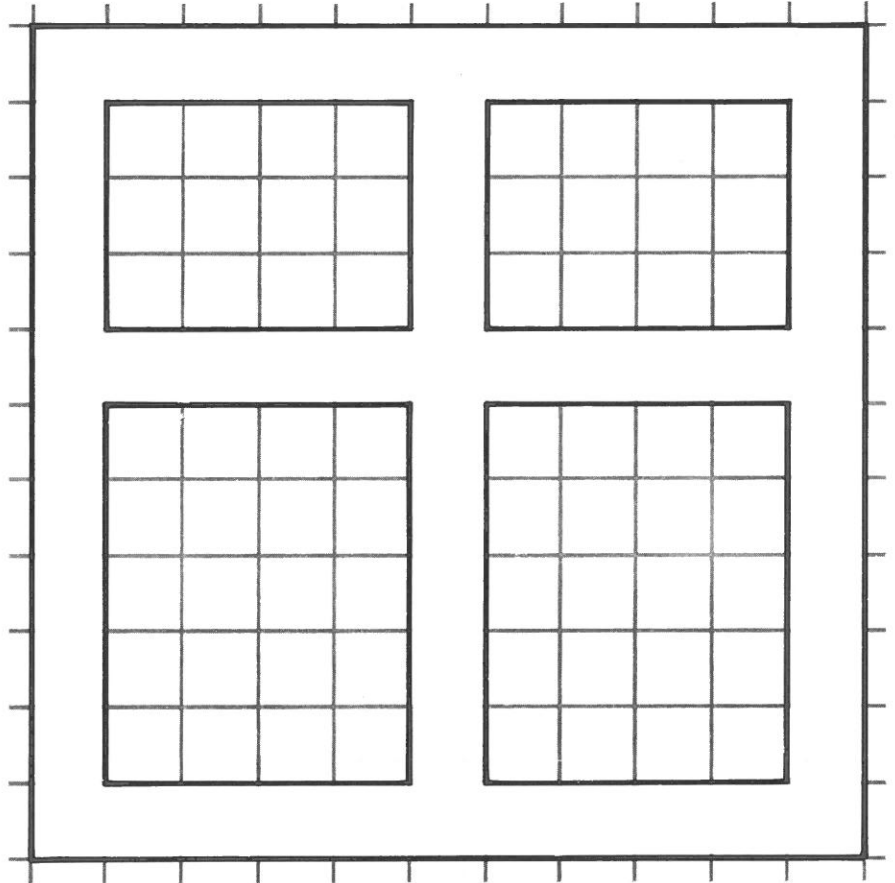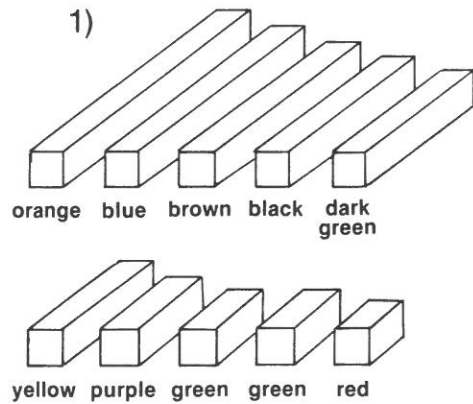
COVERING DESIGNS WITH A SPECIFIC NUMBER OF RODS

1)

Use exactly seven rods to cover this design. Each rod must be a different color. Record the color names on the design to show your solution.
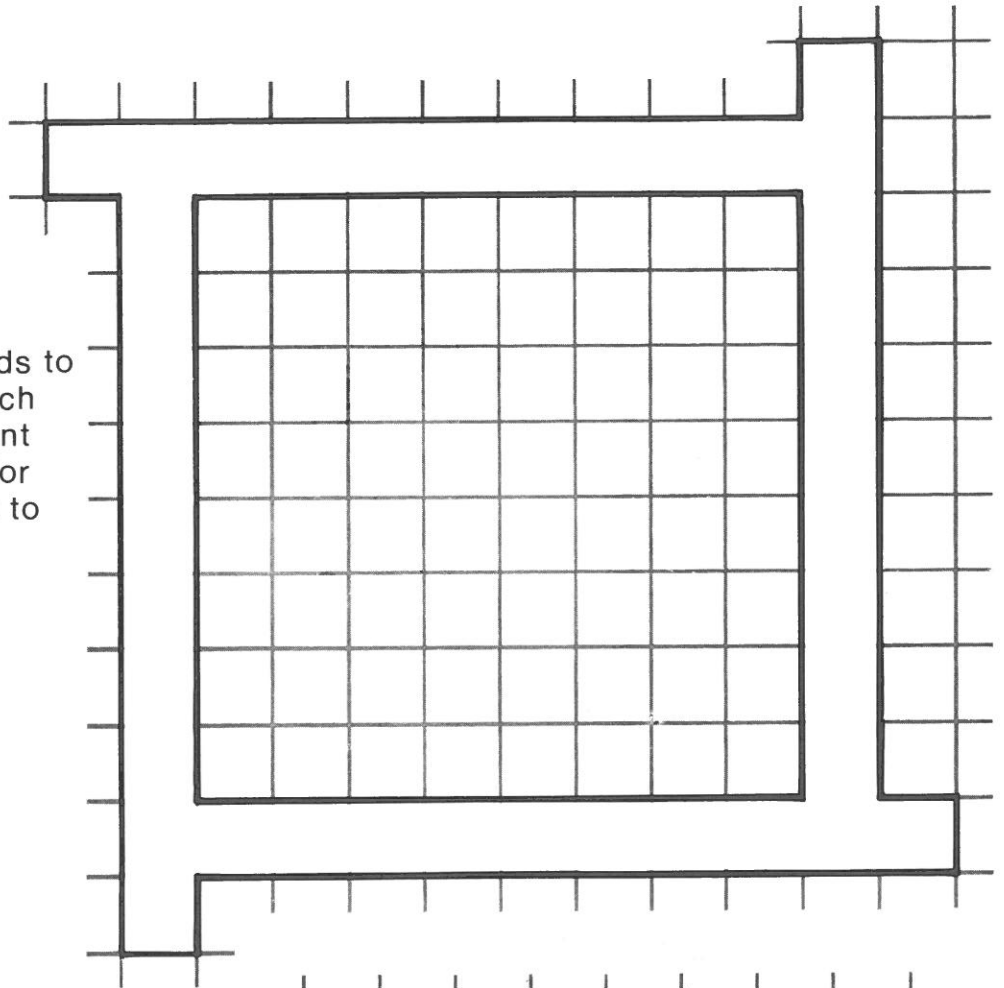
2)

Use exactly eight rods to cover this design. Each rod must be a different color. Record your solution.
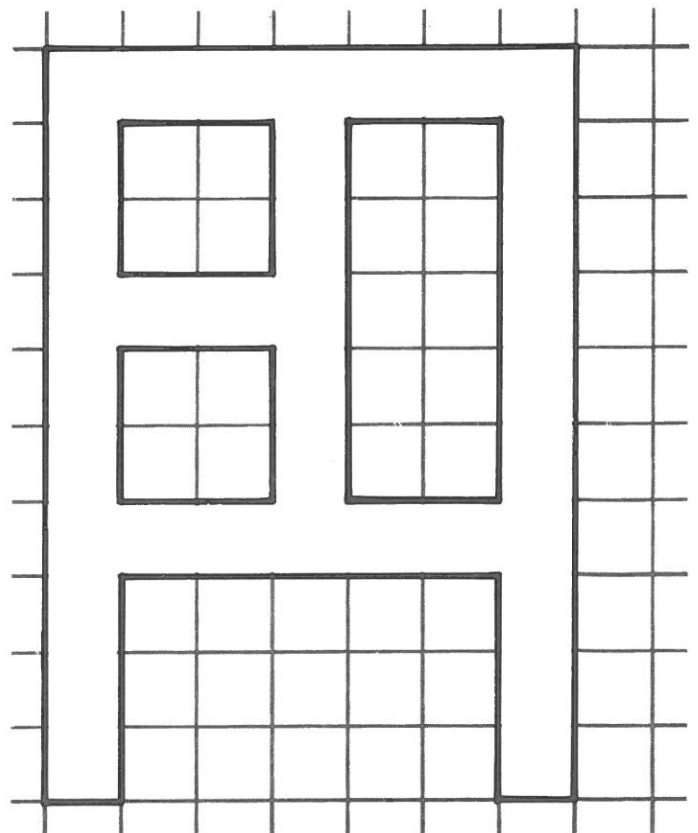
COVERING DESIGNS WITH A SPECIFIC NUMBER OF RODS

1)

Use exactly eight rods and only two colors to cover this design. Record the color names on the design to show your solution.

2)

Use exactly twelve rods and only three colors to cover this design. Record your solution.

COVERING DESIGNS WITH A SPECIFIC NUMBER OF RODS

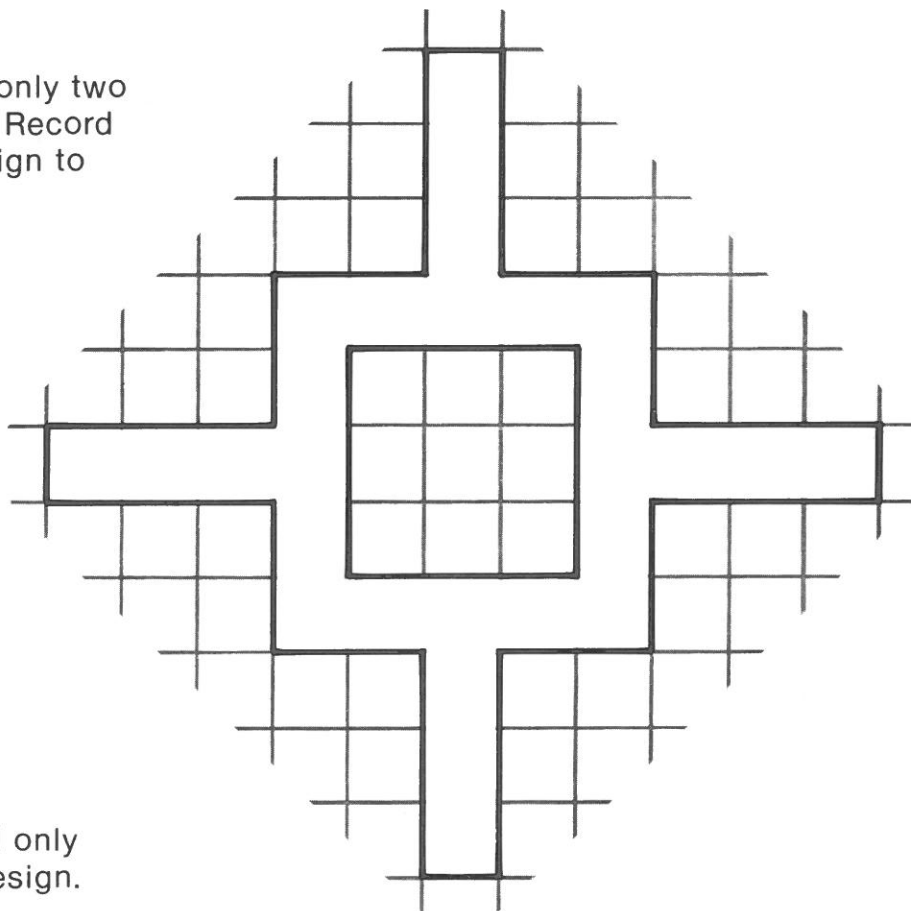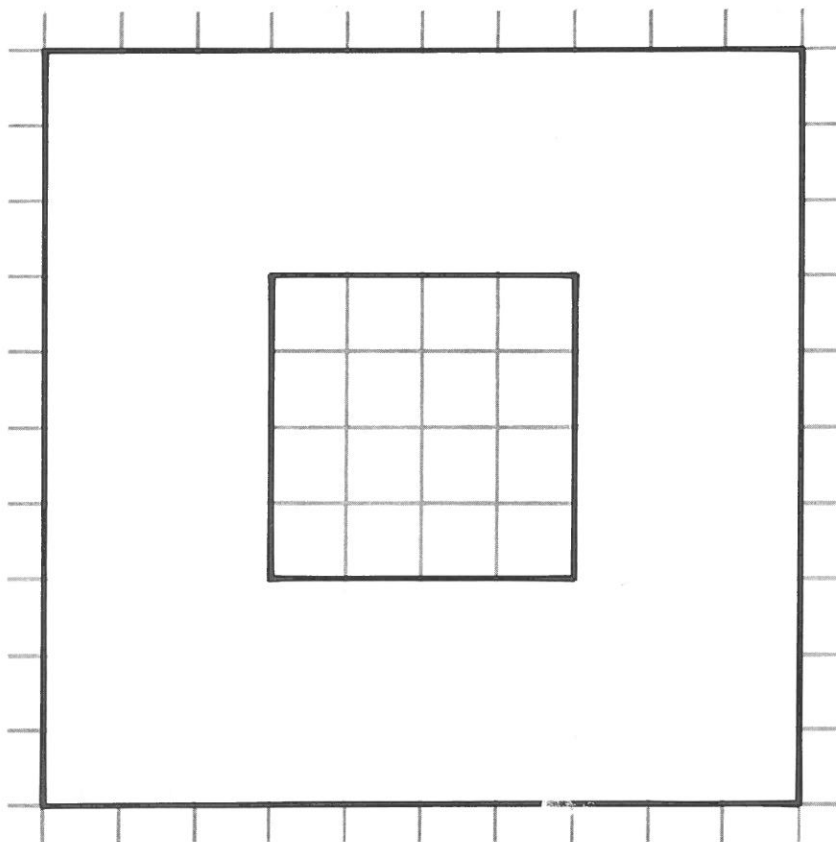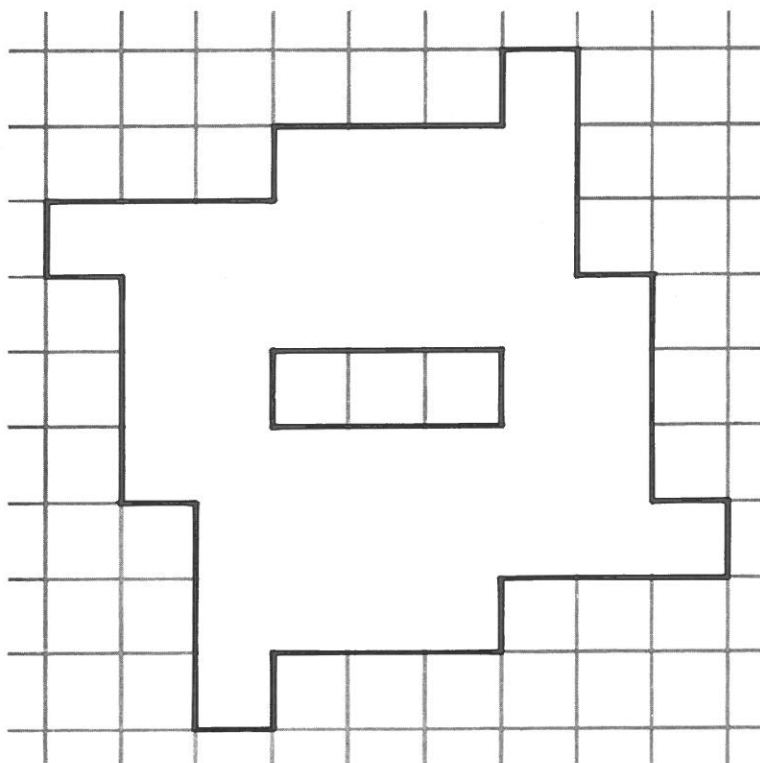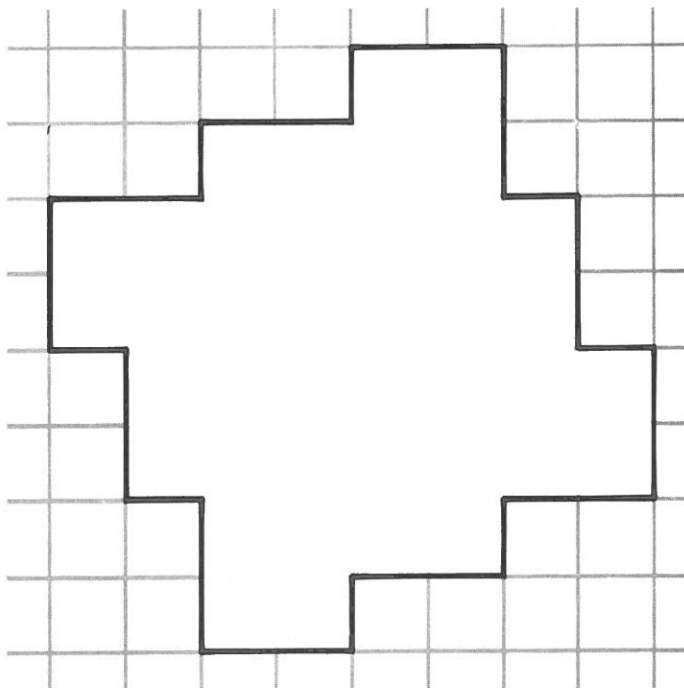**1)**

Use exactly ten rods and only two colors to cover this design. Record the color names on the design to show your solution.

2)

Use exactly fourteen rods and only two colors to cover this design. Record your solution.

DRAWING ROTATED ROD DESIGNS

Make the given design with rods. Rotate the design in a clockwise direction the number of degrees indicated. Then draw the design resulting from each rotation. Compare the original design and the rotated design so that you can visualize the effect of each type of rotation. You may wish to use the Master Cut-Out #2 from page 60 to help you.

Original Design   **Rotated Design**

1)   180°

2)   90°
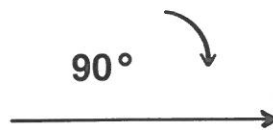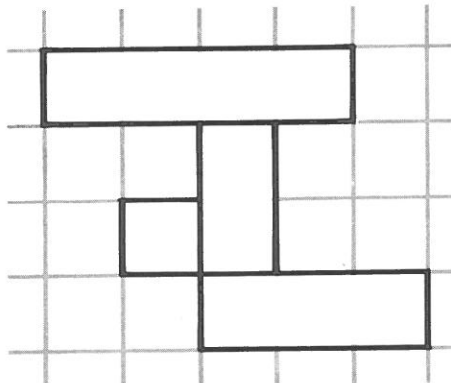
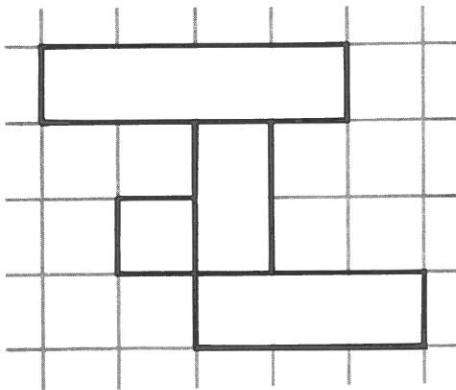3)   270°
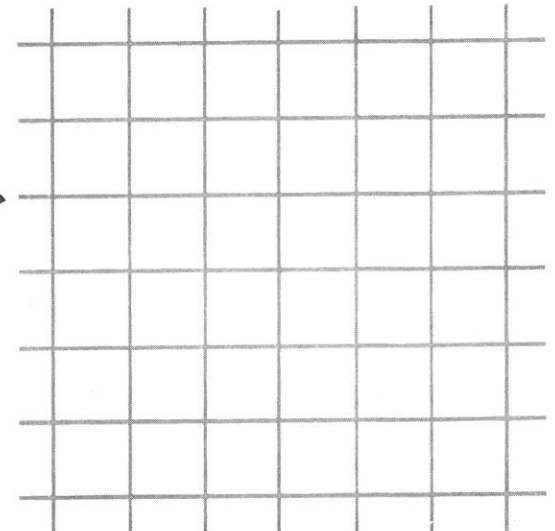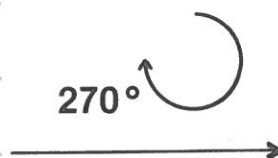
ROTATING DIFFERENT ROD DESIGNS

Make each of the given designs with rods. Rotate the design in a clockwise direction the number of degrees indicated. Then draw the design resulting from each rotation.

1)

180°

2)

270°

3)

90°
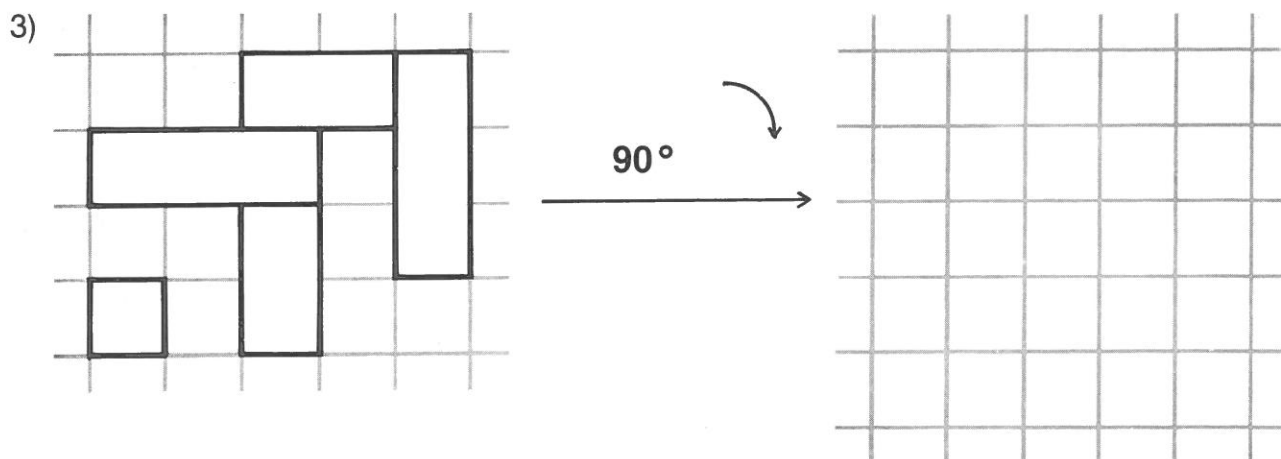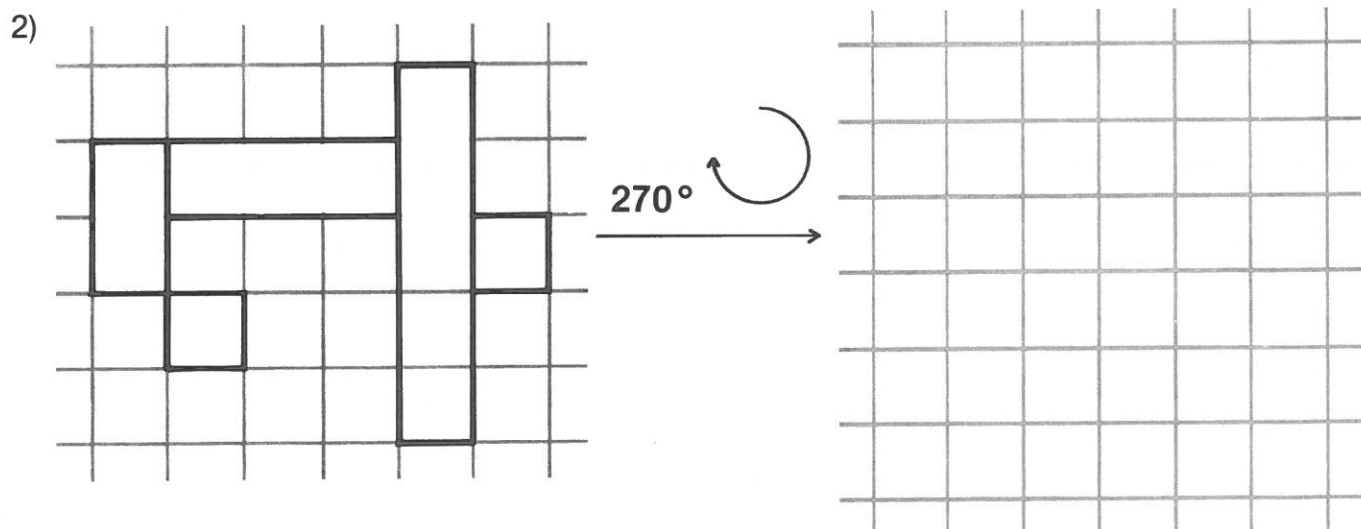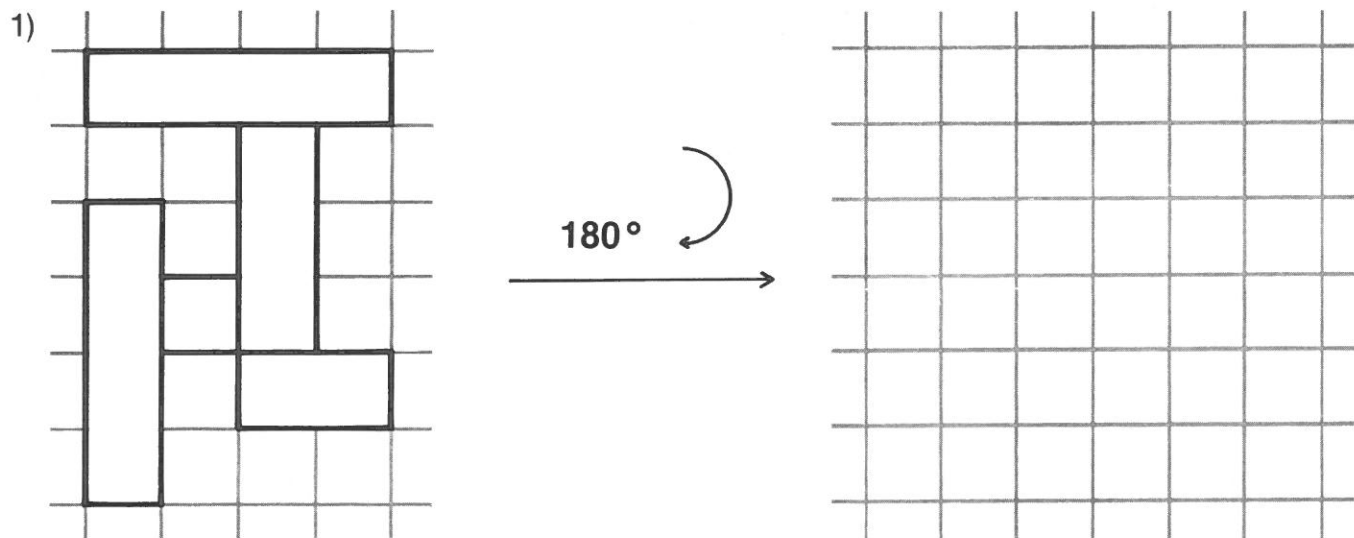
FINDING THE TYPE OF ROTATION

Use rods to make the design shown in the left column. Then name the type of rotation that would produce the resulting design shown in the right column.

Fill in 90° ⤷ , 180° ⤸ , or 270° ↺
in the circle above the arrow.

1)

2)

3)

DRAWING REFLECTED ROD DESIGNS

Make the given designs with rods. Reflect each design as indicated. Then draw the design resulting from each reflection. Compare the original design and the reflected design so that you can visualize the effect of each type of reflection. You may wish to use the Master Cut-out #4 from page 60 to help you.
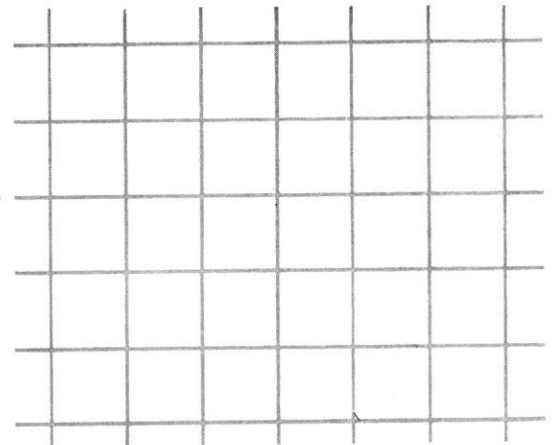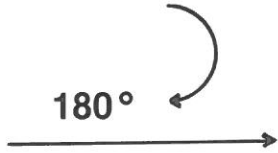
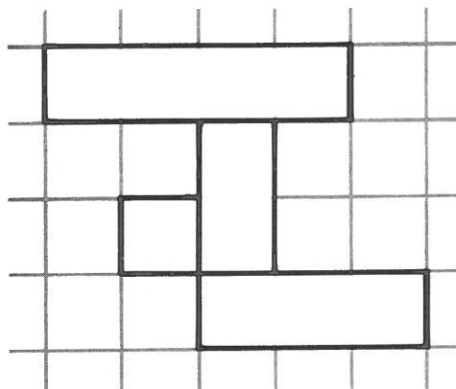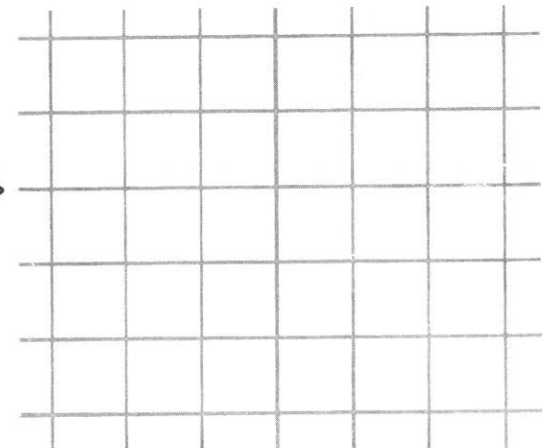**Original Design**   **Reflected Design**

1)

H ------>

2)

L

3)

V

4)

R

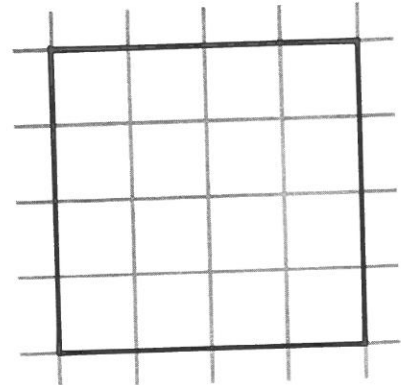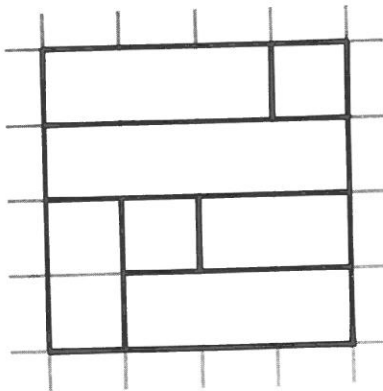# REFLECTING DIFFERENT ROD DESIGNS

Make the given designs with rods. Reflect each design as indicated. Then draw the design resulting from each reflection.

1)   **V** | →

2)   **R**   /   →

3)   **H** - - - - - →

4)   **L**   \   →
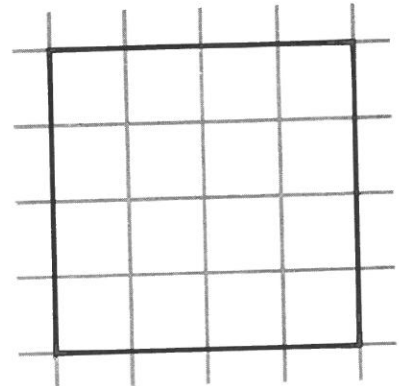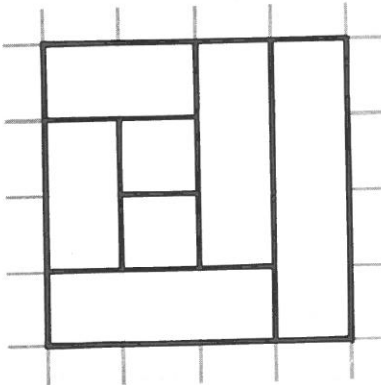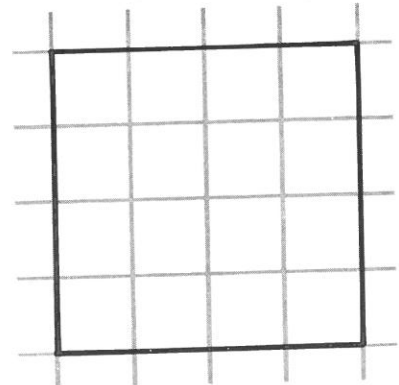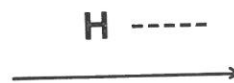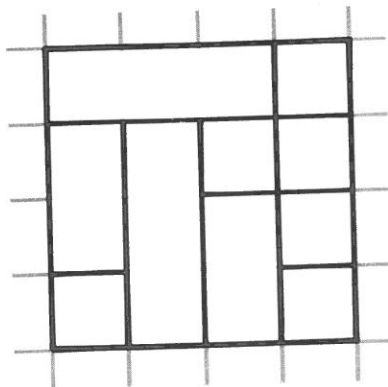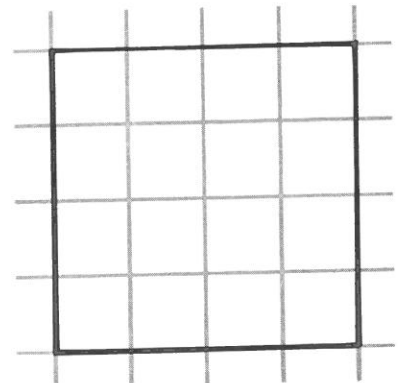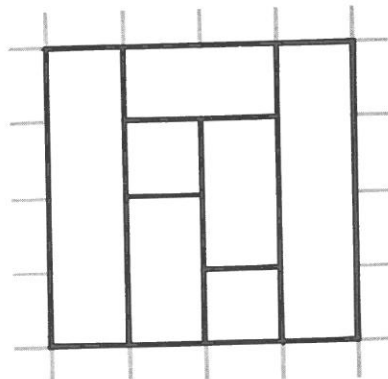
FINDING THE TYPE OF REFLECTION

Use rods to make the design shown in the left column. Then name the type of reflection that would produce the resulting design in the right column.

> **Fill in a V** ⦙ , **H** --- , **L** \ , **or**
> **R** ╱ **in the circle above the arrow.**

1)

2)

3)

SOLVING ROTATION AND REFLECTION ANALOGY PUZZLES

Solve each of these three analogy puzzles using your rods. Fill in the circles with the code: 90° ↷; 180° ↶), 270° ↺ , V ┊ , H┄┄, L↘, or R↗. Find and draw the missing resulting design in the space provided.

1)

2)

3)

SOLVING ROTATION AND REFLECTION ANALOGY PUZZLES

Solve each of these three analogy puzzles using your rods. Fill in the circles with the code: 90° ↘, 180° ↩, 270° ↺, V ┊ , H---; L ↘, or R ↗. Find and draw the missing resulting design in the space provided.

1)

2)

3)

SOLVING ROTATION AND REFLECTION ANALOGY PUZZLES

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1)

2)   3)

SOLVING ROTATION AND REFLECTION ANALOGY PUZZLES

Solve each of these analogy puzzles using your rods. Fill in the circles with the code: 90° ↘, 180° ↻, 270° ↺, V ⦙ , H---; L ⬉, or R ⬈. Find and draw the missing resulting design in the space provided.

1)
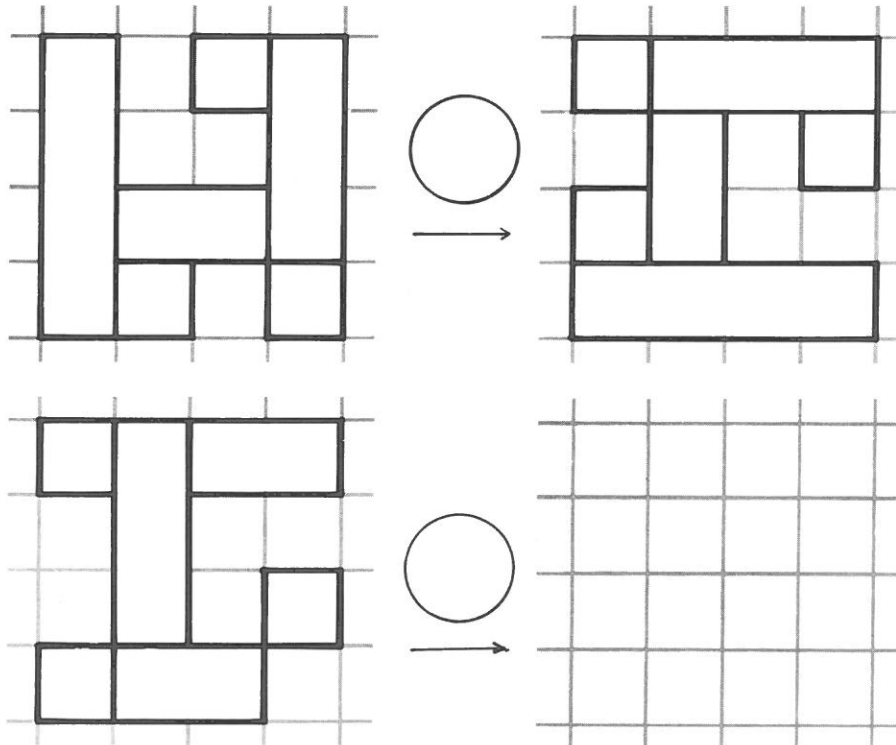
2)
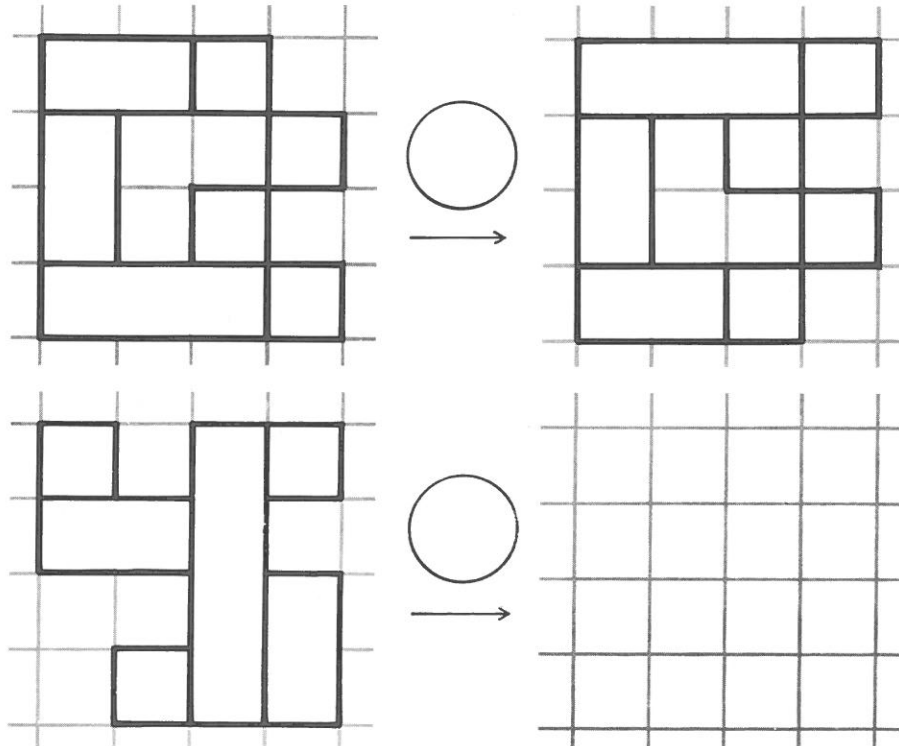
REVERSING ROD ANALOGY PUZZLES

Use your rods to find the missing designs to make these correct analogies. Fill in the circles with the code: 90° ↘, 180° ↻, 270° ↺, V ⋮, H ⋯, L ↘, or R ↗. Draw the missing design in the space provided.

1)
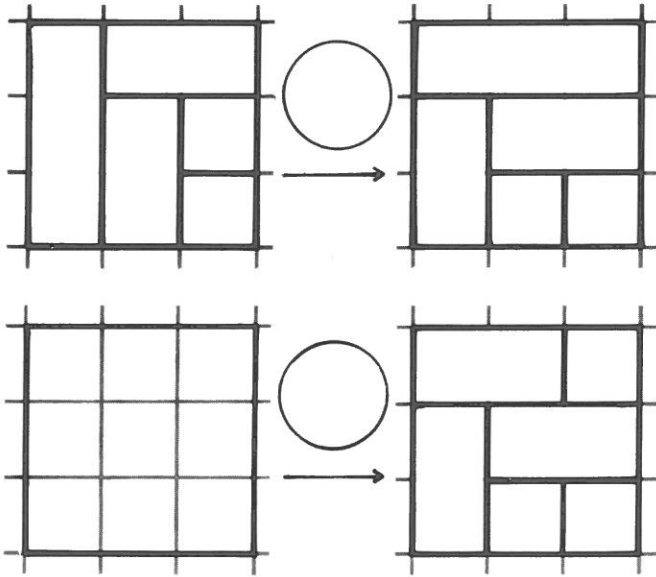
2)
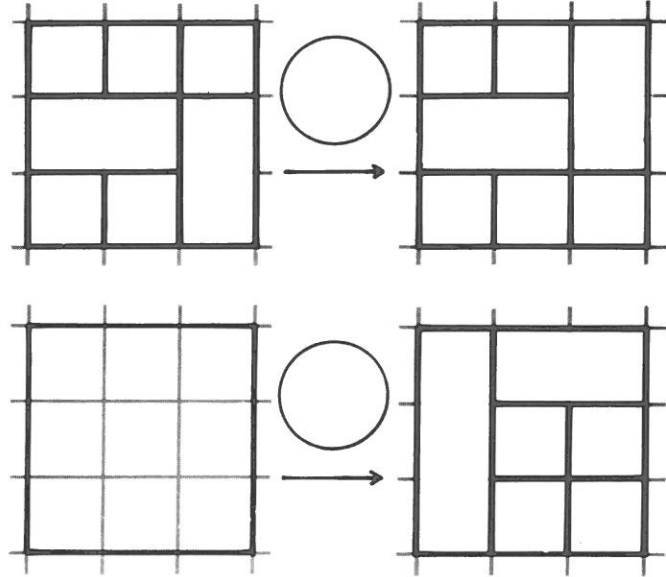
3)

SOLVING A CHAIN OF ROTATIONS AND REFLECTIONS

Use rods to make the design at the top of the page. Follow the arrows and perform the chain of rotations and reflections in succession. Your should end with your original design. If you didn't, try again.

Draw (in pencil) the resulting design at each stage.

You may wish to use the Master Cut-out #7 from page 60 to help you.
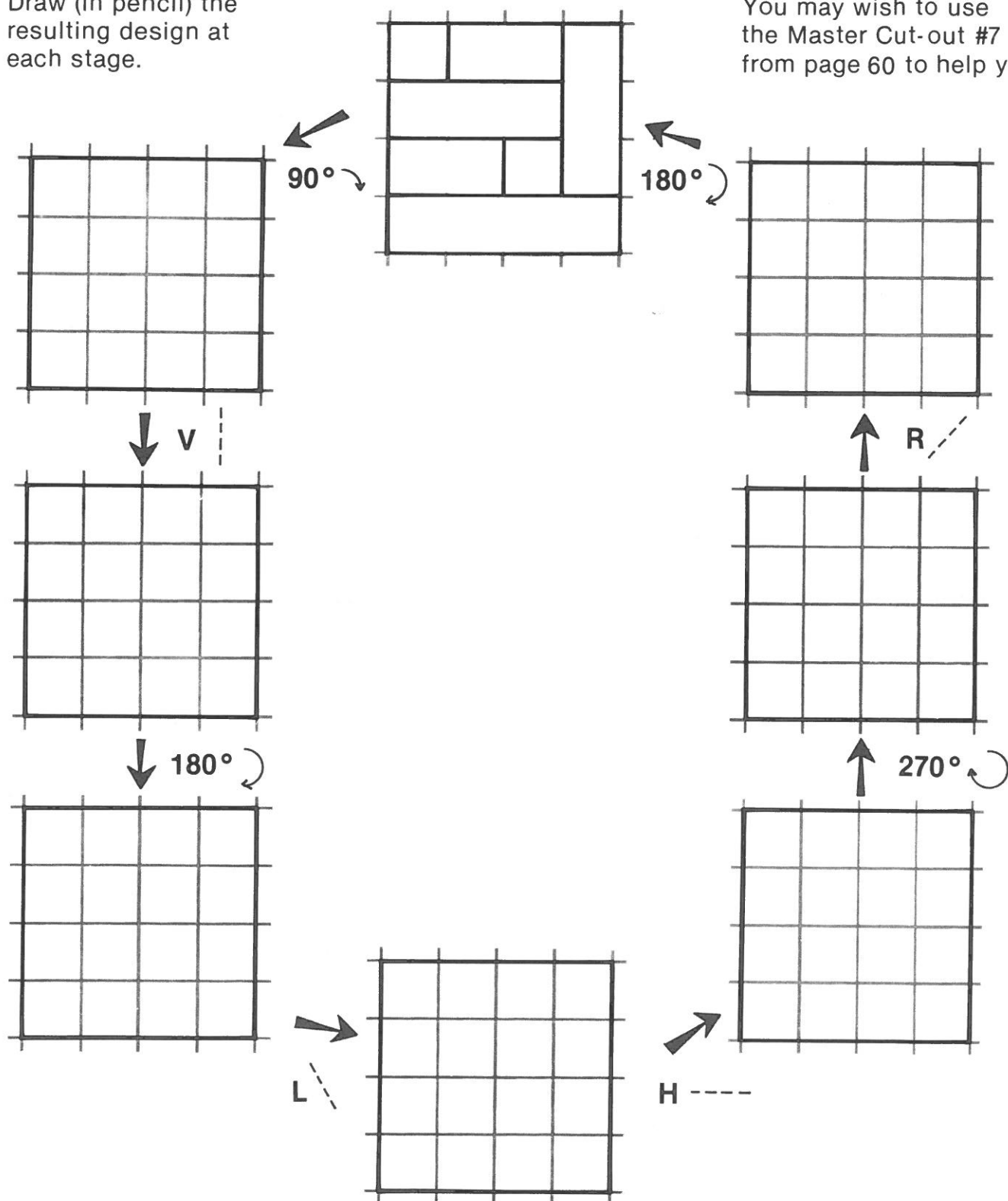
90°

180°

V

R

180°

270°

L

H

SOLVING A CHAIN OF ROTATIONS AND REFLECTIONS

Use rods to make the design at the top of the page. Follow the arrows and perform the chain of rotations and reflections in succession. You should end with your original design. If you didn't, try again.

Draw (in pencil) the resulting design at each stage.

You may wish to use the Master Cut-out #9 from page 60 to help you.

H

H

270°

180°

L

R

R

V
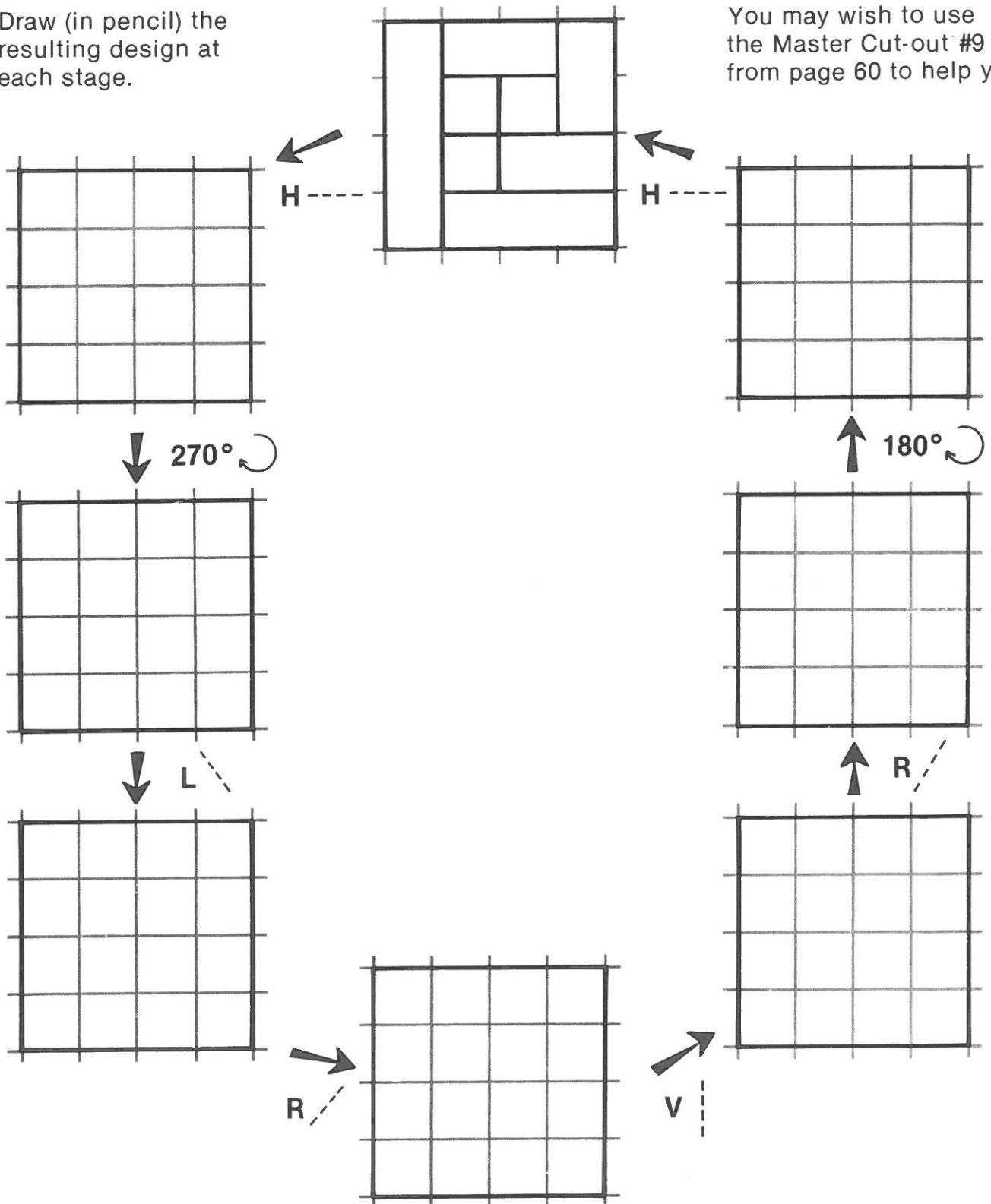
DRAWING THREE-DIMENSIONAL ROD DESIGNS IN TWO DIMENSIONS

1) Build this three-dimensional rod design using 2 light green, 2 red, and 1 white rod.
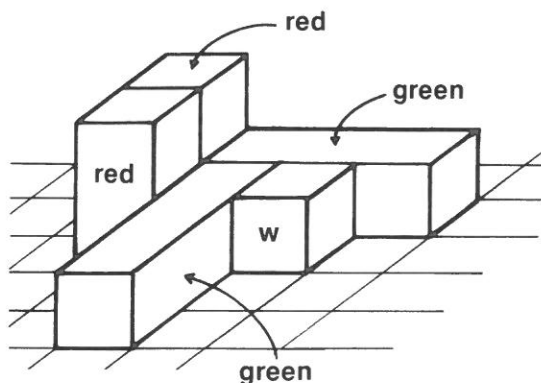
Draw the top view. The front and side views are given.
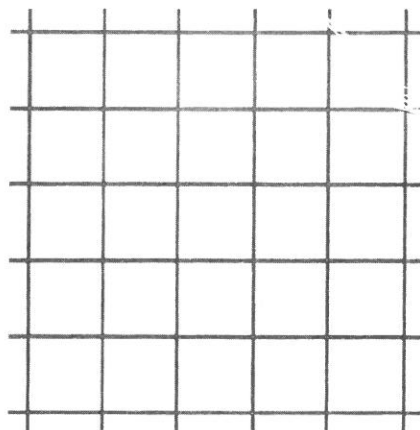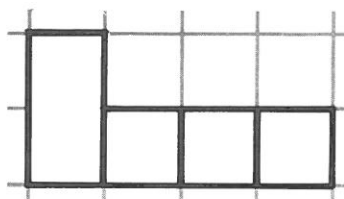
Top View

Front View
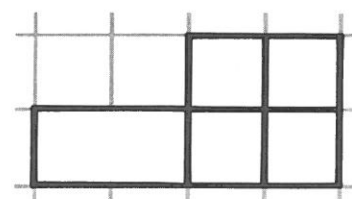
Side View

2) Now build this three-dimensional rod design using 1 purple, 2 light green, 2 red, and 2 white rods.
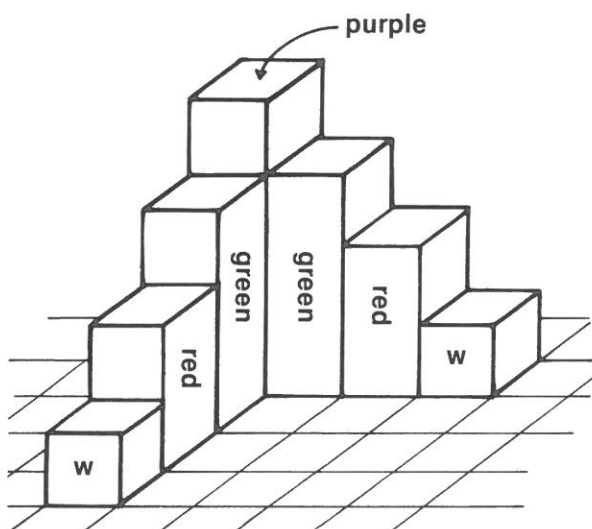
Draw the front view. The top and side views are given.
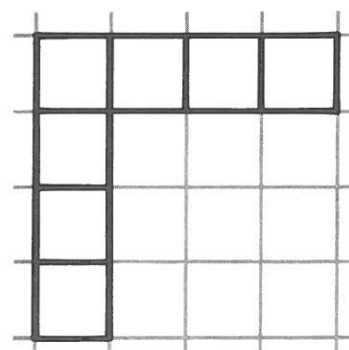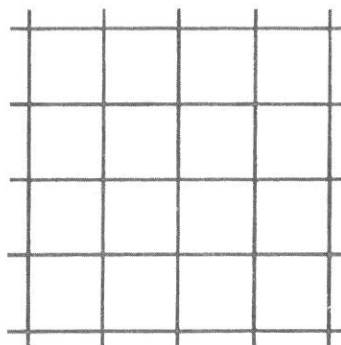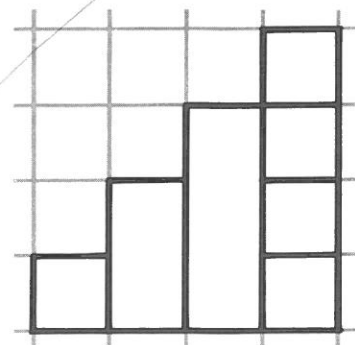
Top View

Front View

Side View

DRAWING THE THREE VIEWS OF A THREE-DIMENSIONAL ROD DESIGN

1) Build this three-dimensional rod design using 1 purple, 1 light green, 4 red, and 2 white rods.
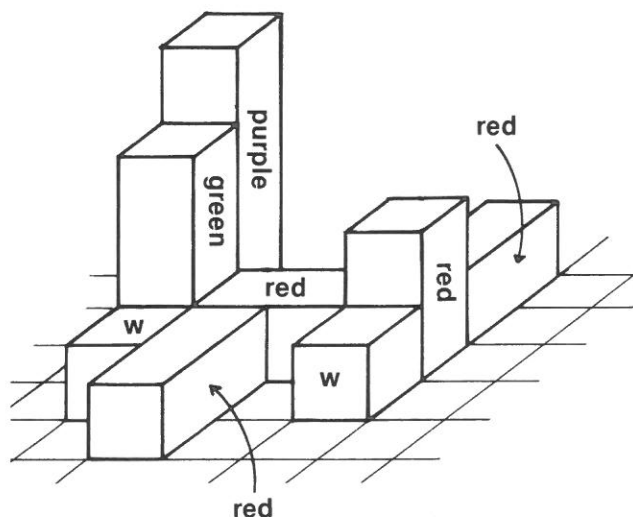
Draw the top, front, and side views in the spaces provided.
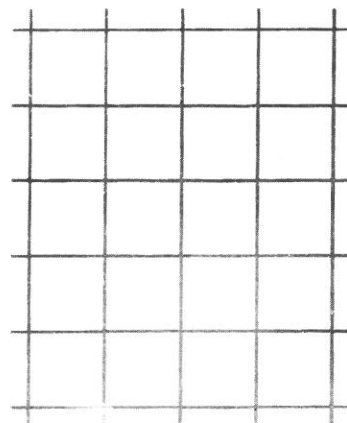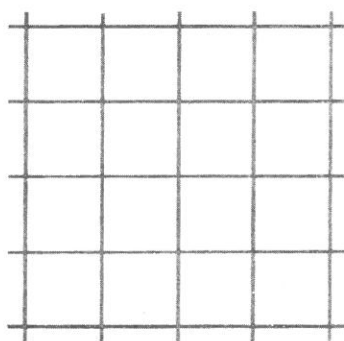
Top View

Front View
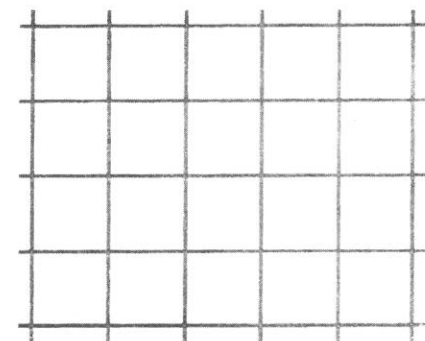
Side View

2) Build this three-dimensional rod design using 1 purple, 2 light green, 1 red, and 3 white rods.
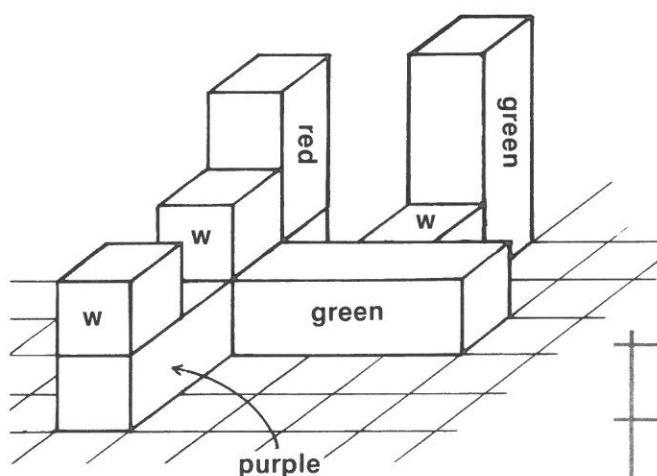
Draw the top, front, and side views in the spaces provided.
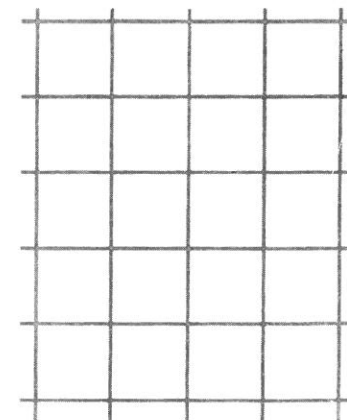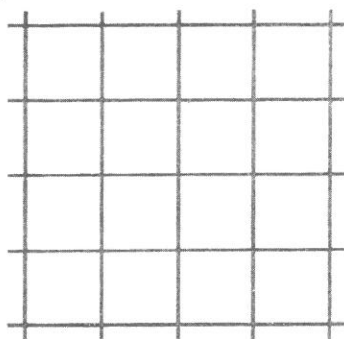
Top View

Front View
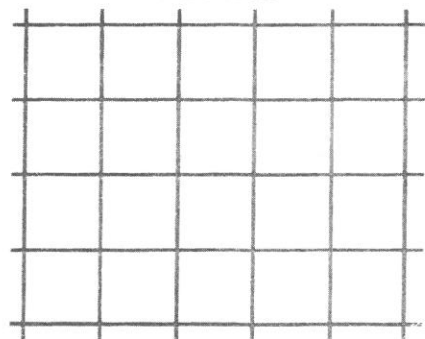
Side View

DRAWING THE THREE VIEWS OF A THREE-DIMENSIONAL ROD DESIGN

1) Build this three-dimensional rod design using 1 yellow, 3 red, and 3 white rods.
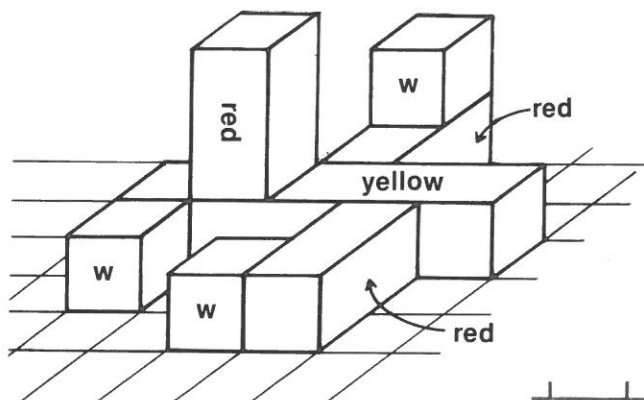
Draw the top, front, and side views in the spaces provided.

Top View

Front View
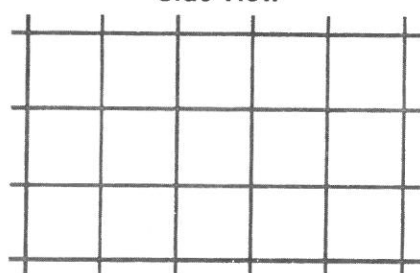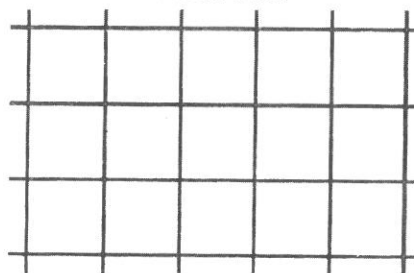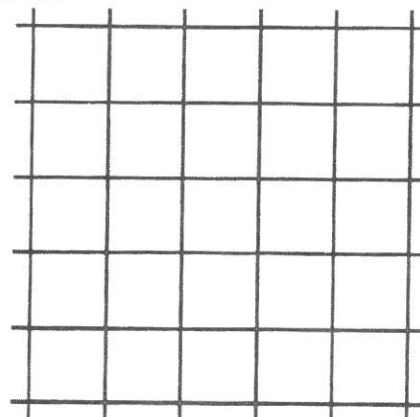
Side View

2) Build this three-dimensional rod design using 1 purple, 1 light green, 2 red, and 3 white rods.

Draw the top, front and side views in the spaces provided.
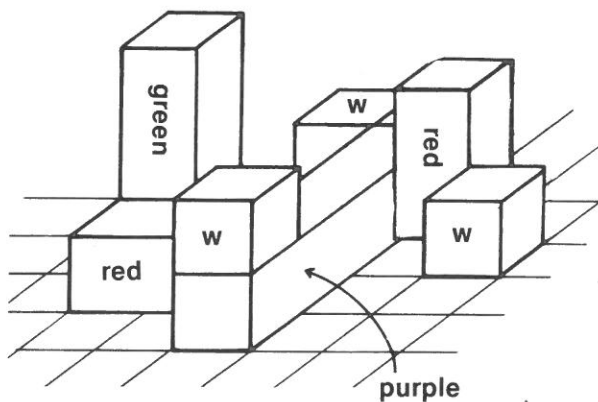
Top View

Front View
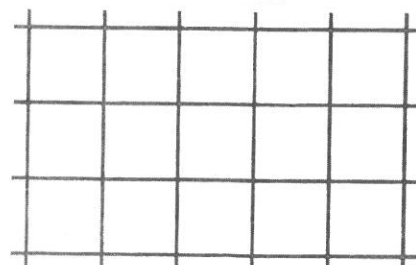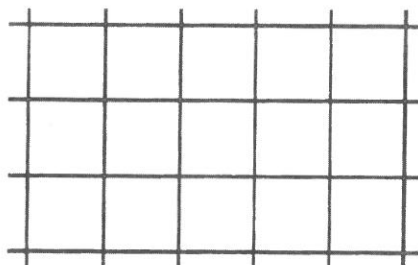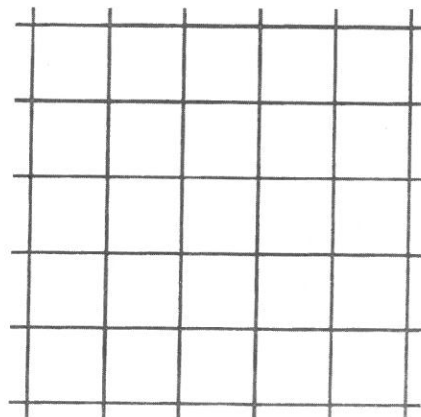
Side View

IDENTIFYING ROD DESIGNS FROM TWO-DIMENSIONAL DRAWINGS

Here are the top, front, and side views of two three-dimensional rod designs. Find which of the three rod designs at the bottom of the page goes with each set of drawings. Cross out the design that will not be used.

Set I:

Top View	Front View	Side View

Set II:

Top View	Front View	Side View
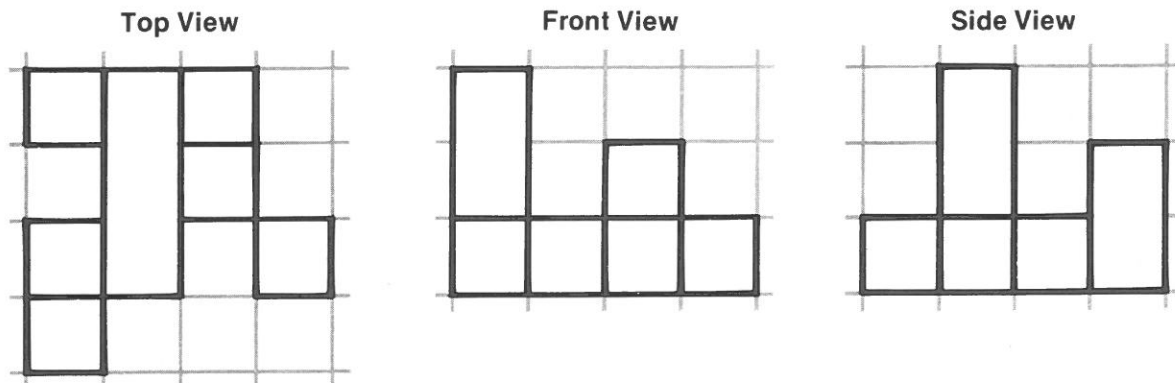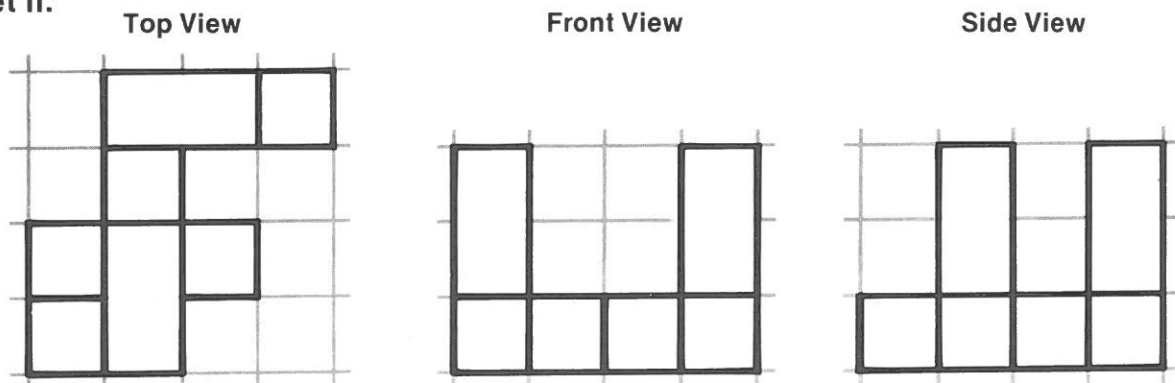
Rod Designs:

Which rod design matches the drawings in Set I? _____

Which rod design matches the drawings in Set II? _____

A.

B.

C.
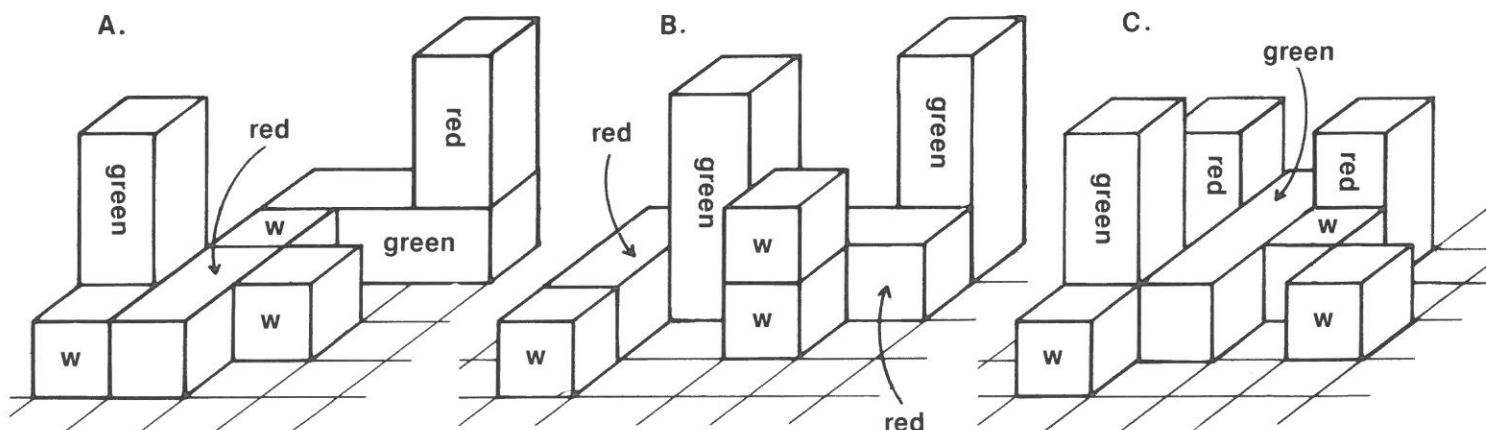
BUILDING ROD DESIGNS FROM TWO-DIMENSIONAL DRAWINGS

Here are the top, front, and side views of three-dimensional rod designs. Build the three-dimensional rod design that goes with each set of drawings. The rods you need are listed. They are the same for each set of drawings.

Set I: Use 1 yellow, 1 purple, 1 light green, 1 red, and 3 white rods.
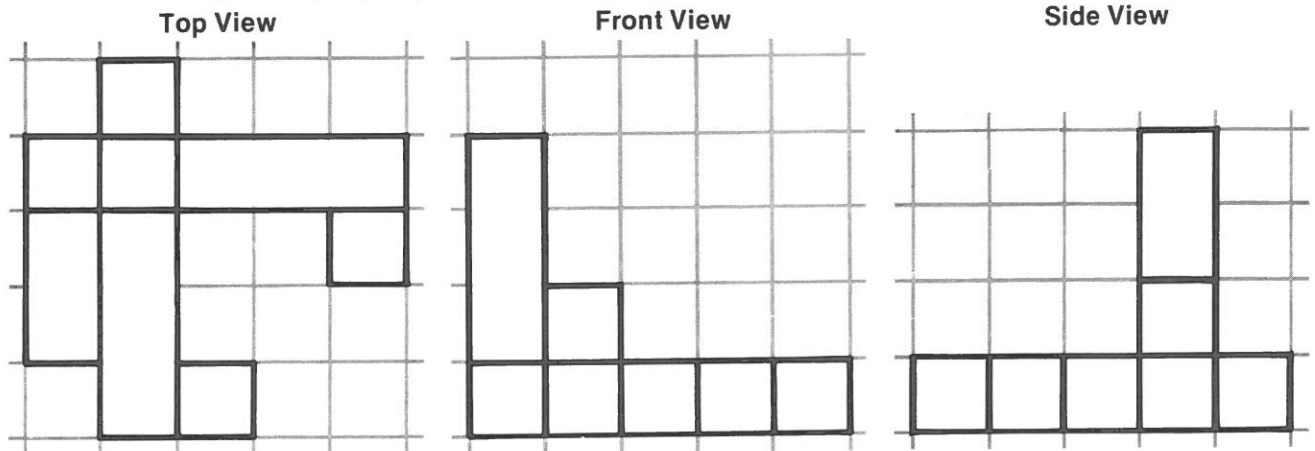
Top View	Front View	Side View

Set II: Use 1 yellow, 1 purple, 1 light green, 1 red, and 3 white rods.

Top View	Front View	Side View

Set III: Use 1 yellow, 1 purple, 1 light green, 1 red, and 3 white rods.

Top View	Front View	Side View

Spatial Problem Solving with Cuisenaire Rods © 1983 Cuisenaire Co. of America, Inc.
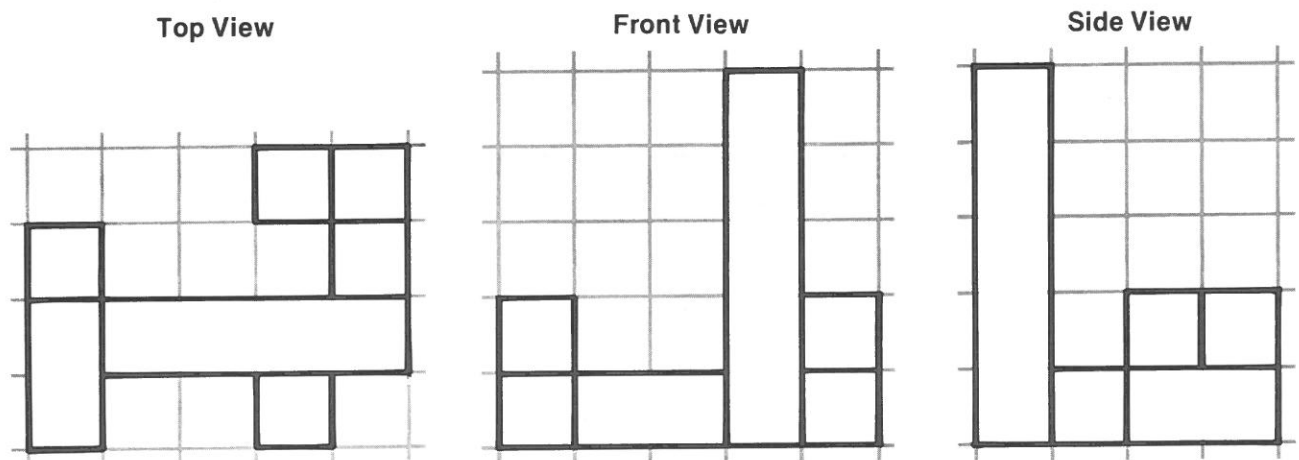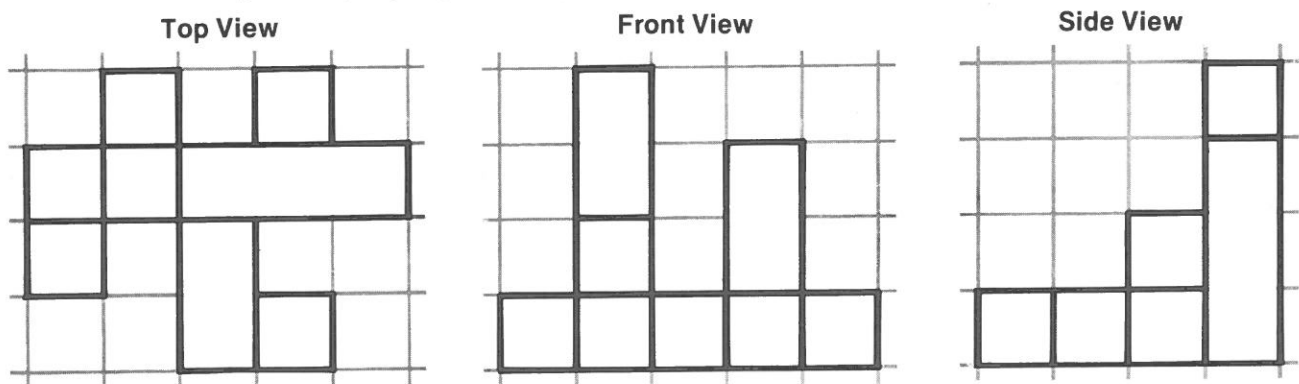
SELECTED ANSWERS AND COMMENTS

Pages 1-14: General Comments

These space-filling activities involve three types of problems: covering designs with one rod of each color, covering designs with a given set of rods, and covering designs with a specific number of rods. The third type, Pages 9-14, is the most challenging and should be preceded by ample experience with the first two types. However, the level of difficulty of particular designs within each type of problem may vary from individual to individual. Students should be given as much time as they need on each problem and should be encouraged to move on if a particular design gives them trouble. They can come back to that design at a later time. Some students may do these tangram-like activities with speed and ease; while others will require much time and many tries. Be prepared to find that the students who excel on these tasks may not be the same students who excel on computational tasks, and that some of your numerical whizzes may be stumped here. Once students discover the strategy to place the large rods first, the solutions become easier to obtain.

Some problems have only one solution; whereas others have more than one solution. The initial goal for these problems should be for students to find just one solution for each design. But as students share their work, other solutions will emerge. Many students will become motivated to find all possible solutions. A great deal of fruitful problem solving is involved in reasoning out why a particular design has a unique solution. Students should be encouraged to discuss their arguments with their classmates. There are so many levels on which these designs can be approached that they meet the needs of a variety of students and can be explored several times throughout the year.

Page 1:

Two possible solutions are shown which are quite different — in one case, the orange rod is placed horizontally; and in the other, vertically. This design has a great deal more flexibility than the one on page 2; hence students should be encouraged to find many ways of solving it.
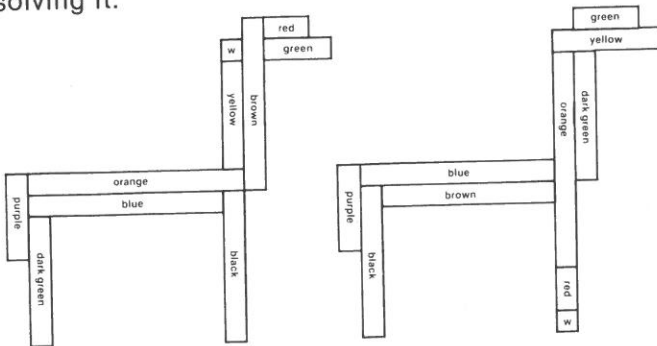
Page 2:

Two possible solutions are shown which are quite similar. In contrast to page 1, this design offers less op-portunity to change rods from a horizontal to a vertical orientation. The number and variety of solutions are limited. If students have difficulty finding even one solution they should move on to page 3 which may seem easier to them and come back to this design at a later time.
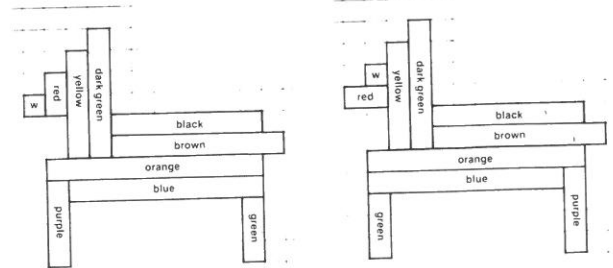
Page 3: 1)

This is an easy problem for most students since there are many solutions and a great deal of flexibility in the placement of the rods. Two possible solutions are shown.
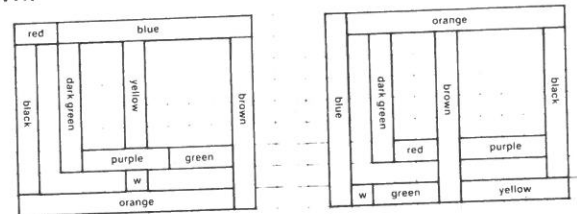

2)

In order to get a solution for this design, the longest rods (orange, blue, and brown) can be placed only in the locations shown, illustrating how crucial it is to place the big rods first. The small rods offer some flexibility.
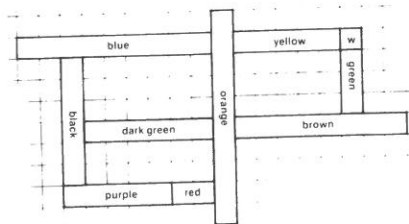
One possible solution is shown.

Page 4: 1)

This design has only one solution. Students should be encouraged to discuss why this is so. Such discussions enhance problem solving skills.
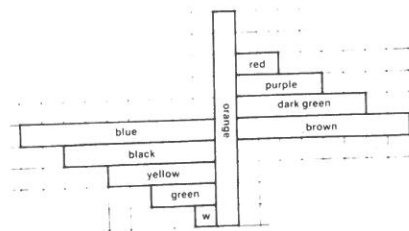
Page 10: 2)

Two solutions are given. It should be noted that they involve the same choice of rods which are placed in different arrangements to produce the design. An interesting topic for discussion is why the choice of rods is unique even though their placement is not.
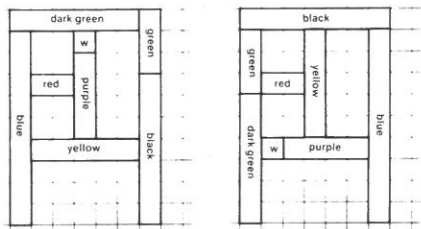
Page 11: 1)

The choice of rods is unique, but several arrangements are possible. Two possible solutions are shown.
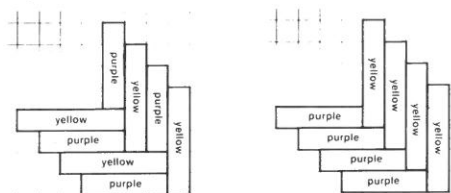
2)

Two different solutions with different sets of rods are possible.

This solution in unique.   This set of rods can be modified in their placement.

Page 12: 1)

Two possible solutions are given for each design.

2)
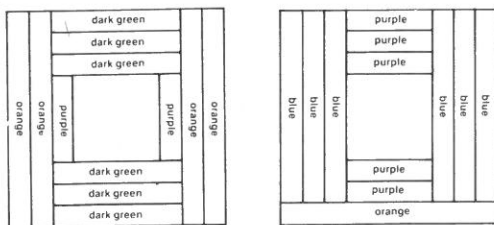
Page 13: 1)

The solutions to this design lend themselves to patterning. For example, there can be 4 of each of the three colors — red, green, and dark green, placed in the following configuration.
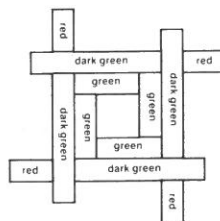
Another solution involves 2 dark green, 4 yellow, and 6 red rods, placed as follows. Again the patterns are interesting.
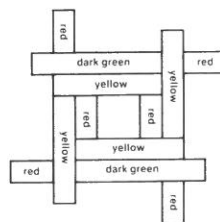
There are other solutions as well. No solution can use orange, blue, or brown rods. The students should argue why this is so.
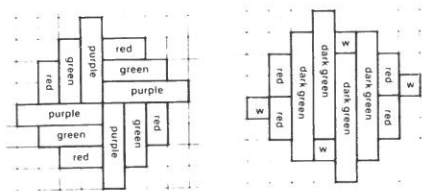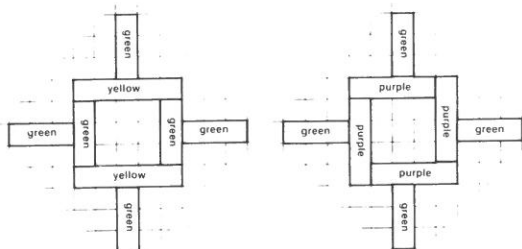
2)

Students may find this problem difficult to solve, as the solution is unique. They may be able to reason that it is impossible to use orange, blue, or brown rods. Once these "long" rods are eliminated, "small" rods like white, red, and green also need to be eliminated. Even with these insights, the choice and placements of the rods remain a challenge.
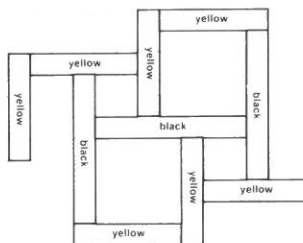
Page 14: 1)

The solution to this design is unique. Students can argue that it is impossible to use orange, blue, brown, or black.
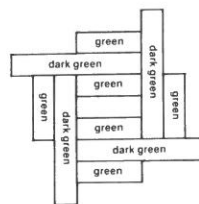
A different problem may be posed for this same design. "Use exactly twelve rods and only two colors". The solution this time is 6 yellows and 6 reds.

2)

Two possible solutions are given to what may seem like another challenging problem because of the restrictions. Students should be encouraged to reason out what rods are impossible to use and to find other solutions besides these two.

Page 21:
This page requires no written work. The goal is to teach students four types of reflection: vertical, V⫶ ; horizontal, H --- ; left diagonal, L ⟍ ; and right diagonal, R ⟋ . Students will find it physically difficult to reflect the actual rod designs (unless they glue the rods together). Hence a Master Cut-out #3 is provided. The picture of the design should be cut out. For the reflection cutouts, it is necessary to trace the outlines of the rods on the back side of the cut-out. The best way to do this is to hold the cut-out up to the light. Students should physically do the reflections with the cut-out.
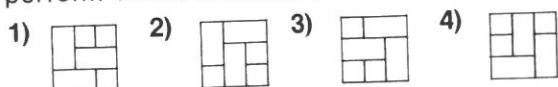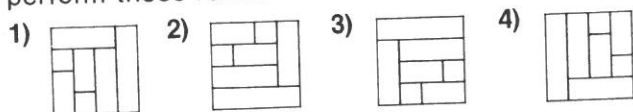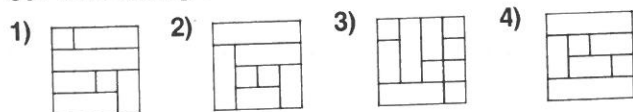
Page 22:
Students should use a copy of the Master Cut-out #4 to perform these reflections.

1)   2)   3)   4)

Page 23:
Students should use a copy of the Master Cut-out #5 to perform these reflections.

1)   2)   3)   4)

Page 24:
Students are asked to reflect four different designs. Some students will already be able to visualize the resulting designs; others may wish to make their own cut-outs using the centimeter graph paper on page 59.

1)   2)   3)   4)

Page 25:
1) V ⫶   2) R ⟋   3) L ⟍   4) H -----

Page 26:
1) H --- 2) V ⫶   3) R ⟋

Pages 27-40: General Comments
The activities in this section combine and apply the concepts of rotations and reflections which were introduced in the previous two sections. It should be noted that pages 15-26 are prerequisite to these pages which include analogies and chains of motions.
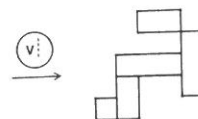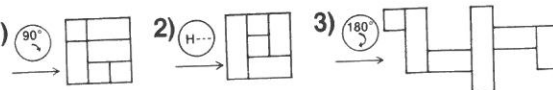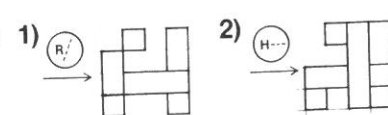
Students may already be familiar with the notion of an analogy. The interpretation here is based on rod designs related either by one of the types of rotations or one of the types of reflections:

   "The first design is to its resulting design
   as the second design is to what?"
Pages 27-40 give a wide variety of experience with analogies. Pages 35 and 36 ask students to reverse the process and find the second design, given the resulting design.

The chains are simply a sequence of motions devised so that if done correctly, they end with the original design. Master Cut-outs are provided to help students

physically carry out the motions. The self-checking nature of these puzzles motivates students to "stay with" these problems even though they require several steps.

Page 27:

Page 28: 1)   2)   3)

Page 29: 1)   2)   3)
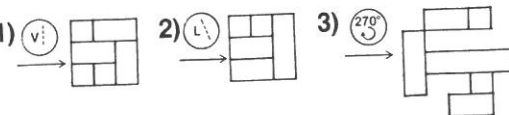
Page 30: 1)   2)   3)
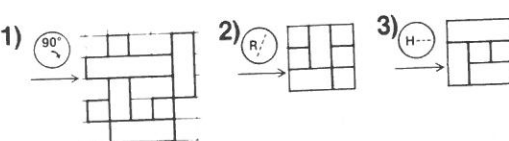
Page 31: 1)   2)   3)
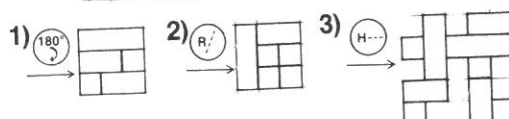
Page 32: 1)   2)   3)
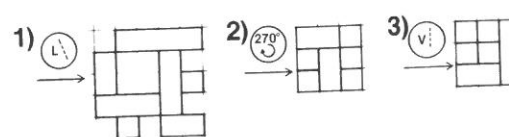
Page 33: 1)   2)
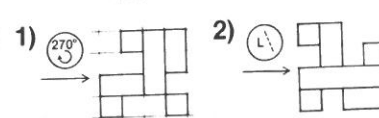
Page 34: 1)   2)

Page 35:
The missing designs this time are on the left side of the arrows. These analogy puzzles require reverse thought processes. "What second design would produce the final design?"

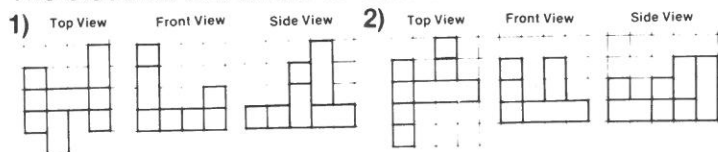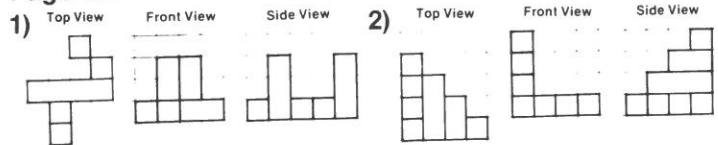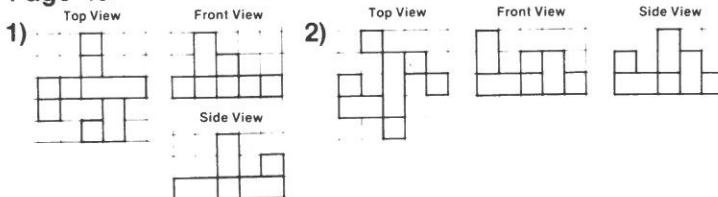1)   2)   3)
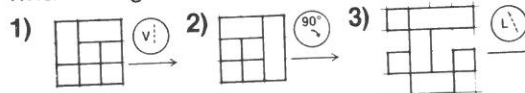
Page 36:
As with page 35, the missing designs are on the left side of the arrows.

1)   2)   3)
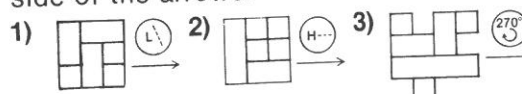
Page 37:
Students may need to try this chain more than once before they get it right; hence they should use a pencil rather than a pen to record their work at each stage. Some students may be able to visualize the rotations and reflections, but most students would benefit from

Page 44:

The students are asked to draw all three views.

1) Top View · Front View · Side View **2)** Top View · Front View · Side View

Page 45:

1) Top View · Front View · Side View **2)** Top View · Front View · Side View

Page 46:

1) Top View · Front View **2)** Top View · Front View · Side View

Side View

Page 47:

This matching task is a preparation for students to be able to build the three-dimensional design for a given set of two-dimensional top, front, and side views.
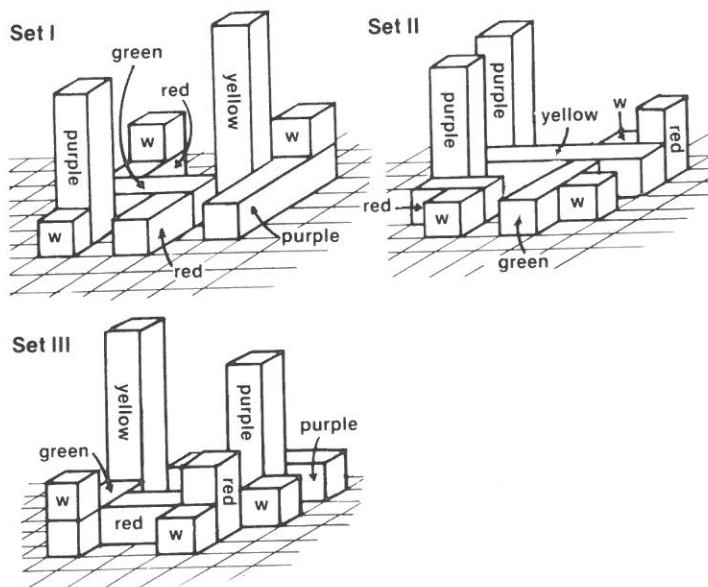
Set I: C   Set II: B

Page 48:

As with Page 47, this matching task is a preparation for students to go from two-dimensional drawings to the three-dimensional design they would generate.

Set I: C   Set II: A

Page 49:

This page has the more difficult task of building the three-dimensional design from the two-dimensional views.

Set I

Set II

Set III

Page 50:

This page also poses the more difficult task of building a three-dimensional design from the two-dimensional views. Students should build each three-dimensional design. Then they should check their answer by drawing the three views for each on centimeter graph paper using the Graph Paper Master provided on page 59.

Set I

Set II

Set III
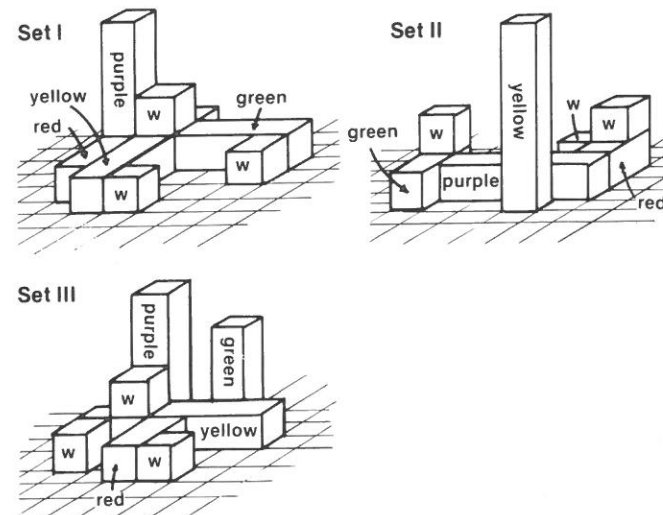
Page 51:

These three rod designs are very challenging since they have open spaces. Only the most spatially adept students will be able to do these. These students may even want to take this activity further by pretending that they are futuristic architectural engineers who like to work with open spaces. They will then make their own three-dimensional rod designs as models of buildings of the future. Then on graph paper, they should draw and label the three views for each design and present them as architectural plans.
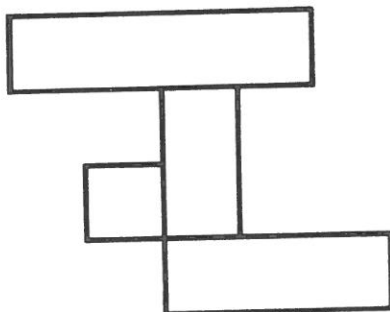
Set I

Set II

Set III
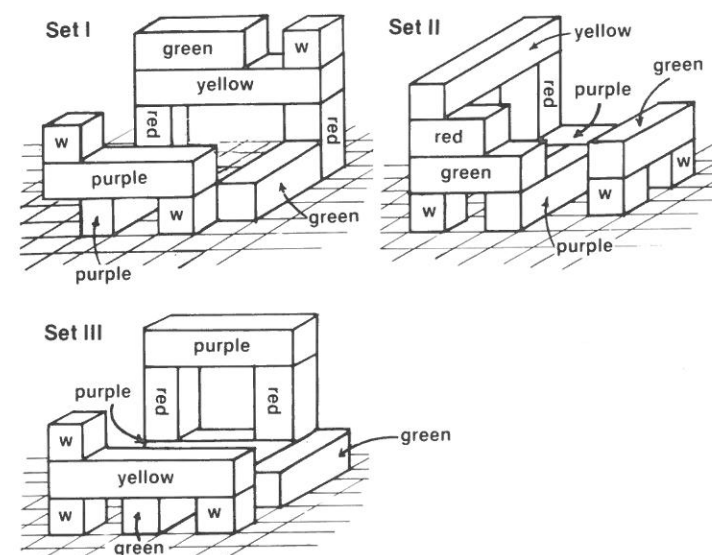
MASTER CUT-OUT SHEET

These rod designs may be cut out to be rotated or reflected as designated on each of the pages lised below. Specific instructions for their use are given in the Selected Answers and Comments Section for each of the pages.
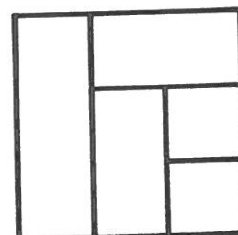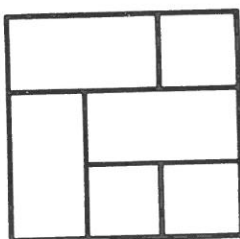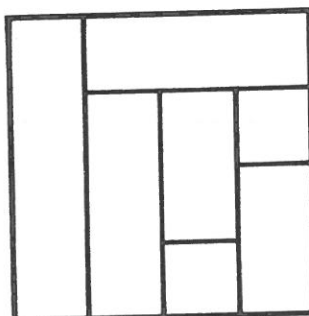
#1. For page 15
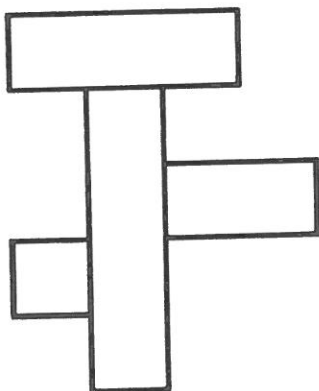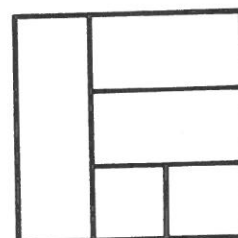
#2. For page 16

#3. For page 21

#4. For page 22

#5. For page 23

#6. For page 37

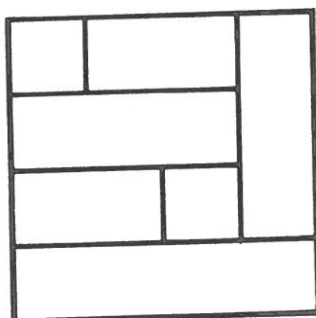**#7. For page 38**

#8. For page 39

#9. For page 40